Background fitting of Fermi GBM observations

Zsolt Bagoly[1,2], Dorottya Szécsi[1] József Kóbori[1], Lajos G. Balázs[1,3] and István Horváth[2],

[1]Eötvös University, Budapest [2] Bolyai Military University, Budapest [3]MTA CSKF Konkoly Observatory, Budapest

Abstract

The Fermi Gamma-ray Burst Monitor (GBM) detects gamma-rays in the energy range 8 keV - 40 MeV. We developed a new background fitting process of these data, based on the motion of the satellite. Here we summarize the result of this method, called *Direction Dependent Background Fitting* (DDBF), regarding the GBM triggered catalog and compare some parameters with the 2-years Fermi Burst Catalog.

Introduction

Fermi has some specific motion in order to survey the sky and catch gamma-ray bursts in the most effective way. However, bursts with an Autonomous Repoint Request (ARR) have a highly varying background, and modeling it with a polynomial function of time is not efficient – one needs more complex, Fermi-specific methods. Here we present the effect of these special moving feature for the measured data, and we introduce some variables based on the position of the satellite related to the Earth and the Sun. We use them together with the time variable to fit a general multidimensional linear function for the background.

Fermi lightcurves with varying background

Fermi uses a complex algorithm in order to optimize the observation of the Gamma-Ray Sky. In Sky Survey Mode, the satellite rocks around the zenith within $\pm 50°$, and the pointing alternates between the northern and southern hemispheres each orbit (3; 2). 12 NaI detectors are placed such a way that the entire hemisphere is observable with them at the same time. The GBM data, which we use in our analysis (called CTIME), are available at 8 energy channels, with **0.064**-second and **0.264**-second resolution (for triggered and non-triggered mode, respectively). The position data is available in **30**-second resolution. This data were evenly proportioned to **0.256**-second and **0.064**-second bins using linear interpolation, in order to correspond to the CTIME data.
Using GBM data, 1-second bins and summarizing the counts in the channels between 11.50–982.23 keV, one can plot a GBM lightcurve as shown in Fig. 1.

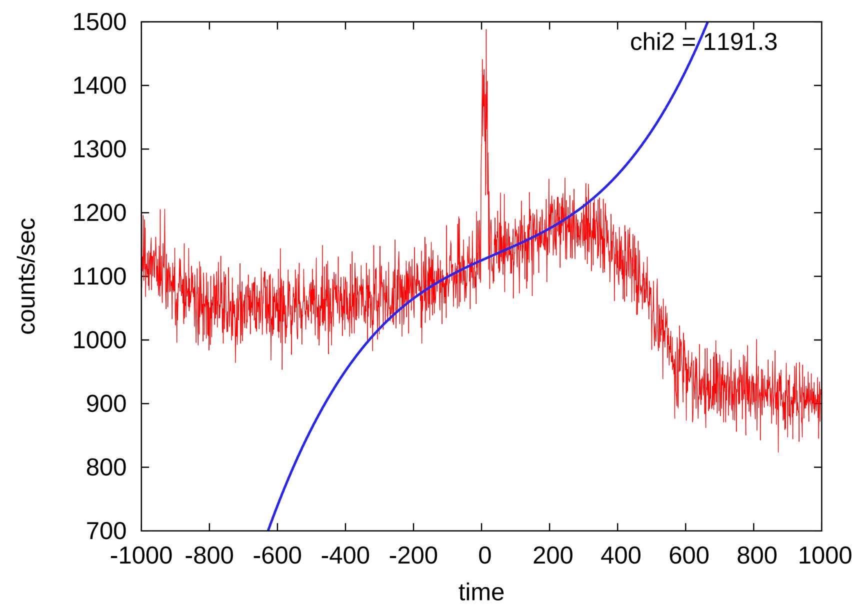

Figure 1: Lightcurve of the Fermi burst 091030613 measured by the 3rd GBM-detector, without any background filtering, with 1-second bins. The grey line is a fitted polynomial function of time of order 3, between -200 and 200 seconds, which does not seem to be a correct model for this whole background: reduced "chi-square" statistics is given in the top right corner. (4).

Direction Dependent Background Fitting (DDBF)

Analyzing ancillary spacecraft and other directional data we have found the following $(\mathbf{x}_i)$ variables, which seem to contribute to the variation of the background: celestial distance between burst and detector orientation, celestial distance between Sun and detector orientation, rate of the Earth-uncovered sky and time.
We use the method of General Least Square for multidimensional fitting of the $\mathbf{y_i}$ counts to the corresponding $(\mathbf{x})$ explanatory variables. The maximum likelihood estimate of the model parameters a_k is obtained by minimizing the quantity of

$$\chi^2 = \Sigma_{i=1}^{N} \left(\frac{y_i - \Sigma_{k=1}^{M} a_k X_k(\mathbf{x}_i)}{\sigma_i} \right)^2 \quad (1)$$

The matrix of A and vector $\mathbf{b}$ are:

$$A_{ij} = \frac{X_j(\mathbf{x}_i)}{\sigma_i}, \quad b_i = \frac{y_i}{\sigma_i}. \quad (2)$$

Minimizing χ^2 leads us to the following equation:

$$\mathbf{a} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}, \quad (3)$$

, where $\mathbf{A}^T$ means the transpose of $\mathbf{A}$, and the expression $(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T$ are called *generalized inverse* or *pseudoinverse* of $\mathbf{A}$.
One cornerstone of the fitting algorithm described above is the definition of the boundaries which divide the interval of the burst and the intervals of the background. In this work, we follow the common method of using user-selected time intervals (5).

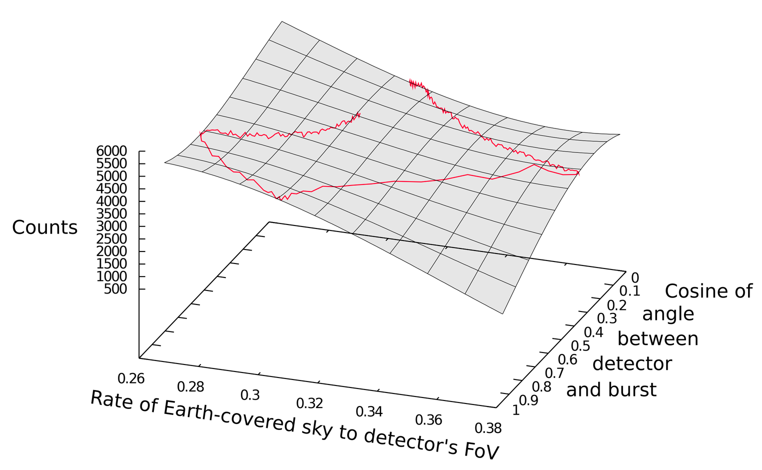

Figure 2: The 2-dimensional hypersurface of 3rd degree fitting to a Fermi lightcurve is shown. The fitted variables $(x_i^{(1)}, x_i^{(3)})$ are along the horizontal axes, while vertical axis represents the counts of the lightcurve y_i (shown by the black curve on the fitted

Eqn. (3) describes a hypersurface, and it is a generalization of fitting a straight line to the data. In the most complicated cases of the ARR backgrounds, however, higher degree of the explaining variables are needed. One can illustrate the lightcurve data y_i and the fitted hypersurface $y(\mathbf{x}_i)$ using 2 variables $x_i^{(1)}$ and $x_i^{(3)}$, both of 3rd degree, on a 3D plot, see Fig. 2. The design matrix of this problem is

$$\mathbf{A} = \begin{pmatrix} x_1^{(1)} & x_1^{(1)2} & x_1^{(3)} & x_1^{(3)2} & x_1^{(1)} \cdot x_1^{(3)} & 1 \\ x_2^{(1)} & x_2^{(1)2} & x_2^{(3)} & x_2^{(3)2} & x_2^{(1)} \cdot x_2^{(3)} & 1 \\ \dots \\ x_N^{(1)} & x_N^{(1)2} & x_N^{(3)} & x_N^{(3)2} & x_N^{(1)} \cdot x_N^{(3)} & 1 \end{pmatrix}. \quad (4)$$

For calculating the pseudoinverse of the design matrix $\mathbf{A}$ we used Singular Value Decomposition (SVD): $\mathbf{A} = \mathbf{USV}^T$. Using $\mathbf{U}$ and $\mathbf{V}$, the pseudoinverse of $\mathbf{A}$ can be obtained as

$$pinv(\mathbf{A}) = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T = \mathbf{VS}^{-1}\mathbf{U}^T. \quad (5)$$

Let us have $y(\mathbf{x}_i)$ as the function of $\mathbf{x}_i = (x_i^{(1)}, x_i^{(2)}, x_i^{(3)}, x_i^{(4)})$ of order 3, so the basis functions $X_k(\mathbf{x}_i)$ (and columns of the design matrix) consist of every possible products of the components $x_i^{(l)}$ up to order 3. That means that we have $M = k_{max} = 35$ basis functions and $a_1, a_2 ... a_{35}$ free parameters. It is sure that we do not need so many free parameters to describe a non-ARR background. Computing the pseudoinverse we need the reciprocal of the singular values in the diagonals of $\mathbf{S}^{-1}$, but if we compute the pseudoinverse of $\mathbf{A}$, the reciprocals of the tiny and not important singular values will be unreasonably huge and enhance the numerical roundoff errors as well. This problem can be solved defining a *limit* value, below which reciprocals of singular values are set to zero.

Model selection

The Akaike Information Criterion (AIC) is a commonly used method of choosing the right model to the data (1). Assume that we have M models so that the kth model has k free parameters ($k = 1...M$). When the deviations of the observed values from the model are normally and independently distributed, every model has a value AIC_k so that

$$\mathrm{AIC}_k = N \cdot \log \frac{RSS_k}{N} + 2 \cdot k \quad (6)$$

, where RSS_k is the Residual Sum of Squares from the estimated model ($RSS = \Sigma_{i=1}^{N} (y_i - y(\mathbf{x}_i, k))^2$), N is the sample size and k is the number of free parameters to be estimated. Given any two estimated models, the model with the lower value of AIC_k is the one to be preferred. Given many, the one with lowest AIC_k will be the best choice: it has as many free parameters as it has to have, but not more.
We loop over the pseudoinverse operation and choose S_{kk} as the limit of singular values in the kth step, and compute the corresponding AIC_k. The number of singular values which minimalize the AIC_k as a function of k will be the best choice when calculating the pseudoinverse.
As an example we analyze the lightcurve of GRB 090113. GRB 090113 is a long burst with T_{90}^{cat}=17.408±3.238 s in the GBM Catalogue, here we show Detector 0 data:

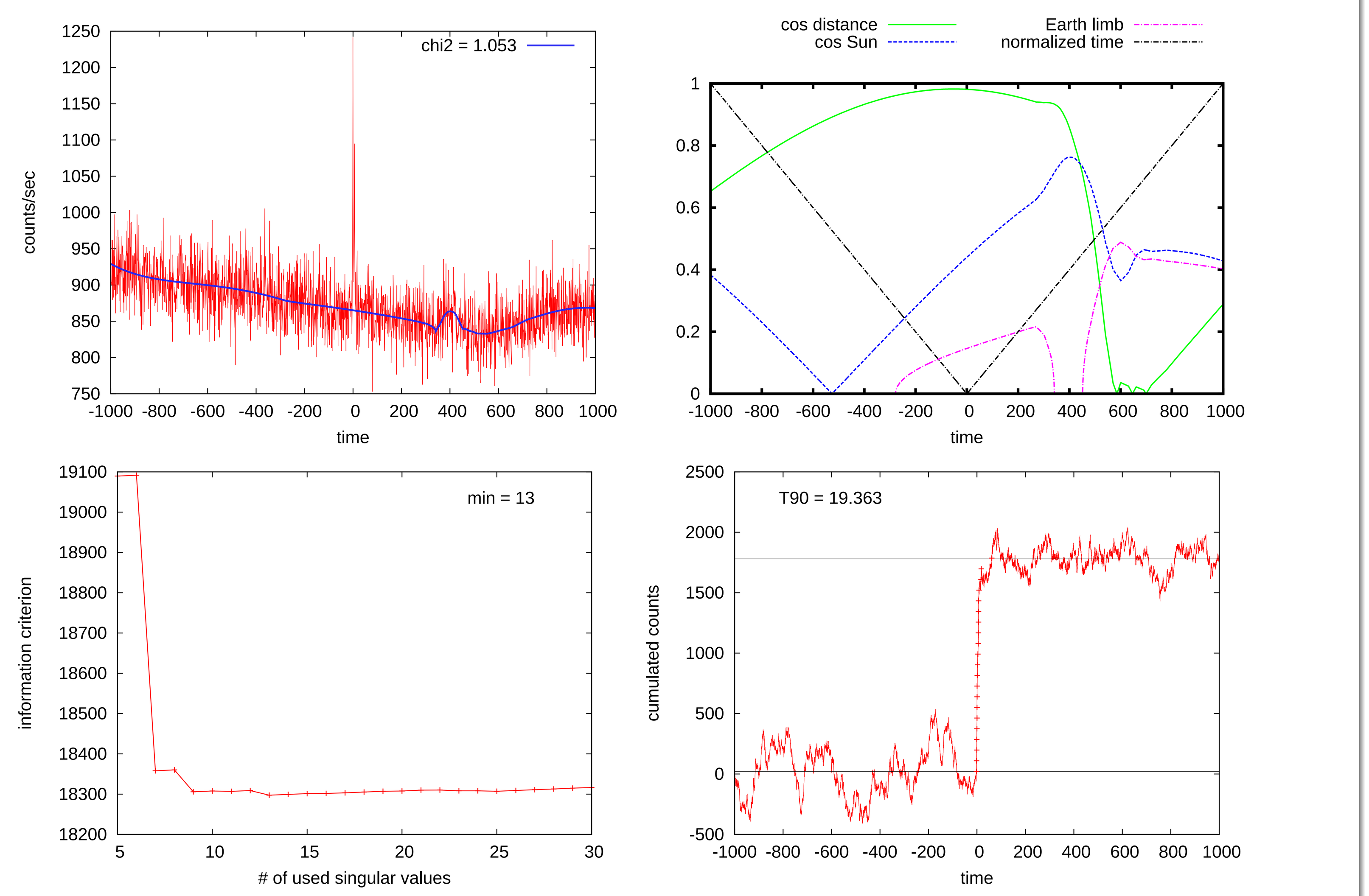

Figure 3: *Up left:* Lightcurve of the Fermi GRB 090113778 measured by the triggered GBM-detector '8', and the fitted background with a grey line. Burst interval (secs): [-20:40]. *Up right:* Underlying variables (absolute values). *Down left:* Akaike Information Criterion. A model with 12 singular values was selected. *Down right:* Cumulated lightcurve of GRB 090113778. $T_{90}=19.679^{+10.883}_{-6.421}$ sec.

Fig. 3 has some extra counts around 400 and 600 seconds. Both of them can be explained with the variation of the underlying variables, that is, the motion of the satellite. These are not GRB signals!.

Error analysis

The DDBF method is too complicated to give a simple expression for the error of T_{90} using general rules of error propagation. We therefore repeated the process for 1000 MC simulated data. Distribution of the Poisson-modified T_{90} and T_{50} values are shown in Fig. 4 for GRB091030613.

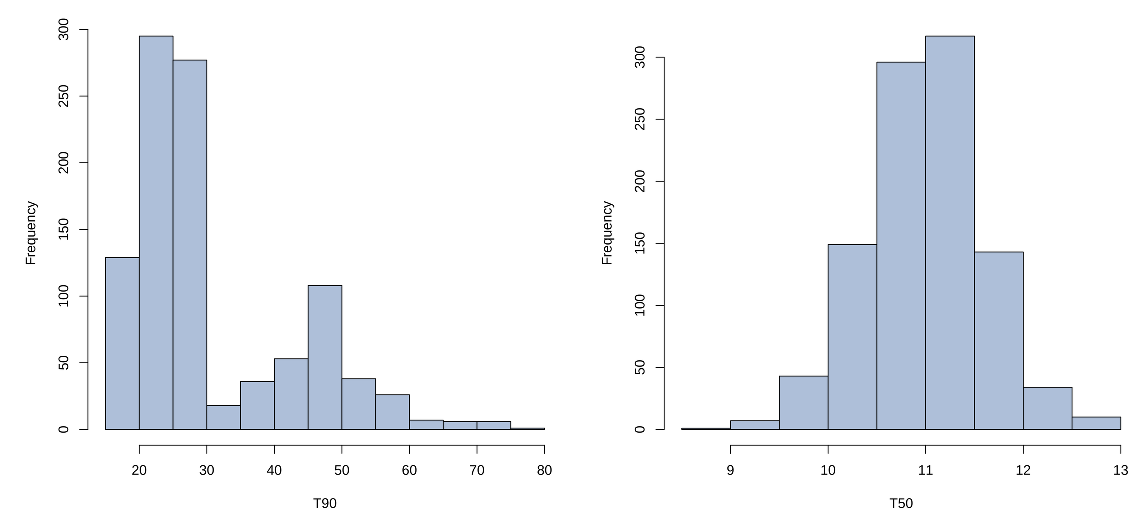

Figure 4: Distibution of the T_{90} (left) and T_{50} (right) values obtained from the MC simulated data for Fermi burst 091030613.

Fig. 4 shows two significant peaks around 22 and 47 seconds. The first peak at 22 seconds corresponds to the measured T_{90} value. However, in some cases of the Poison noise simulation, the measured T_{90} value is systematically longer: that is because this burst has a little pulse around 47 seconds (see Fig. 1.). There is no sign of this second peak in the T_{50} distribution, as it is more robust.

Burst	$T_{90}(s)$	errors (s)		$T_{90}^{catalog}(s)$	$T_{50}(s)$	errors (s)		$T_{50}^{catalog}(s)$
081009360	176.228	**+1.357**	**−9.477**	176.191	15.852	**+3.006**	**−2.350**	25.088
090102122	29.756	**+2.971**	**−1.198**	26.624	10.859	**+0.531**	**−0.556**	9.728
090113778	19.679	**+10.883**	**−6.421**	17.408	6.408	**+0.498**	**−0.344**	6.141
090618353	103.338	**+3.842**	**−6.725**	112.386	22.827	**+2.201**	**−1.530**	23.808
090828099	63.608	**+1.467**	**−1.652**	68.417	11.100	**+0.198**	**−0.194**	10.752
091030613	22.609	**+13.518**	**−4.522**	19.200	10.770	**+0.388**	**−0.424**	9.472
100130777	80.031	**+3.755**	**−3.485**	86.018	32.340	**+0.931**	**−1.363**	34.049

Some results and confidence intervals).

Summary

The commonly used methods are not efficient for most cases of the ARR, so we developed a new technique based on the motion and orientation of the satellite. The DDBF considers the position of the burst, the Sun and the Earth as well. Based on these position information, we computed three physically meaningful underlying variables, and fitted a 4 dimensional hypersurface on the background. Singular value decomposition and Akaike information criterion was used to reduce the number of free parameters. Further research may be required to find a more suitable model dimension reducing criterion.
The DDBF method has the advantage of considering only variables with physical meanings and furthermore, it fits well the whole 2000 sec CTIME data as opposed to the currently used methods. This features are necessary when analyzing long GRBs and precursors, where motion effects influence the background rate sometimes in a very extreme way. Therefore, not only Sky Survey, but ARR mode GRB's can be analyzed, and a possible long emission can be detected.

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contact: zsolt.bagoly@elte.hu