Dependence of the optical brightness on the gamma and X-ray properties of GRBs

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Abstract

The Swift satellite made a real break through with measuring parallel the gamma, X-ray and optical data of GRBs, effectively. Although, the satellite measures the gamma, X-ray and optical properties almost in the same time a significant fraction of GRBs remain undetected in the optical domain. In a large number of cases only an upper bound is obtained. Survival analysis is a tool for studying samples where a part of the cases has only an upper (lower) bound. The obtained survival function may depend on some other variables. The Cox regression is a way to study these dependencies. We studied the dependence of the optical brightness (obtained by the UVOT) on the gamma and X-ray properties, measured by the BAT and XRT on board of the Swift satellite. We concluded that the Duration, gamma Fluence and Peak flux has a significant impact on the distribution of the optical brightness, but the early X-ray flux and the gamma Photon index do not. The reason for the impact of gamma properties on the optical brightness is probably lying in the energetics of the jet launched from the central engine of the GRB and triggers the afterglow in the surrounding interstellar matter.

Introduction

A significant achievement of the Swift satellite is the parallel detection of the physical properties of the gamma ray bursts in the gamma, X-ray and optical domain, measured by the BAT, XRT and UVOT instruments on board of the satellite. Following the alert given by BAT the satellite starts to slew and after reaching the position of the burst the XRT and UVOT make measurements in the X-ray and optical domain, respectively. Although, a significant fraction of the bursts is detected by the XRT as well, it is not the case with the UVOT where at a remarkable fraction of the events only an upper bound of the optical brightness is obtained.

From theoretical point of view the measured optical and gamma properties may be given by completely different phenomena, their observational relationship, if there is any, would be important constraint for the possible models. To study this relationship it would be a serious bias if we take into account only those cases where all properties, i.e. gamma X-ray and optical, are measured. Survival analysis is a way to make use the information which is inherent in the value of the upper bound of the optical brightness.

Cox regression is a tool for studying the dependence of the survival function (a result of the analysis) on some background variables, the covariates (gamma and X-ray properties in our case). In the following we use Cox regression to study the dependence of the distribution of the UVOT detected optical brightness on measured gamma and X-ray properties.

Mathematical Summary

Let we have a *t* stochastic variable with *f(t)* probability density. The *S(t)* survival function is defined by

$$\int_{-\infty}^{t} f(t')dt' = F(t) = 1 - S(t)$$

where *F(t)* means the probability distribution function. Actually, the *S(t)* survival function is its complement (*F(t) + S(t) = 1*). Kaplan and Meier (1958) showed that *S(t)* can be estimated bias free even in the case when some of the values in the t_1, t_2, …, t_n observed sample are only lower bounds (censored). The ratio of *f(t)* to *S(t)* is called the hazard function:

$$h(t) = \frac{f(t)}{S(t)} = -\frac{S'(t)}{S(t)} = -\frac{d}{dt}\log[S(t)]$$

The *h(t)* hazard function characterizes the "risk" that in the *[t,∞]* range (*S(t)* gives its probability) an event will happen in the *[t,t+dt]* interval (its unconditional probability is *f(t)dt*). The hazard function may depend on background variables, covariates. The Cox model (Cox, 1972) assumes that this dependency can be written in the form of

$$\log[h(t)] = \alpha(t) + B_1x_1 + B_2x_2 + \ldots + B_mx_m$$

, where $x_1, x_2, \ldots, x_m$ are the covariates while the *α(t)* arbitrary function and the B_1, B_2, … , B_m constants have to be determined during the procedure of the Cox regression. If all these constants are equal to zero the *α(t)* function identical with the logarithmic hazard function. The value of the constants characterize the strengths of the influence of covariates on the hazard and, consequently on the survival function.

Description of the data

We used for the present analysis the data available in the Swift table (http://swift.gsfc.nasa.gov./docs/swift/archive/grb_table) recorded until the date of 03/03/2012, in particular the V magnitude as a dependent variable, Duration, Fluence, Peak flux, Photon index and early X-ray flux, as covariates in the analysis. Except of the Photon index we used logarithmic values in order to suppress the impact of the outliers on the results.

Since the optical brightness of the GRB afterglow is seriously dimmed by the foreground Galactic extinction we excluded the cases with the latitude of |b| < 15° (|sin(b)| < 0.26).

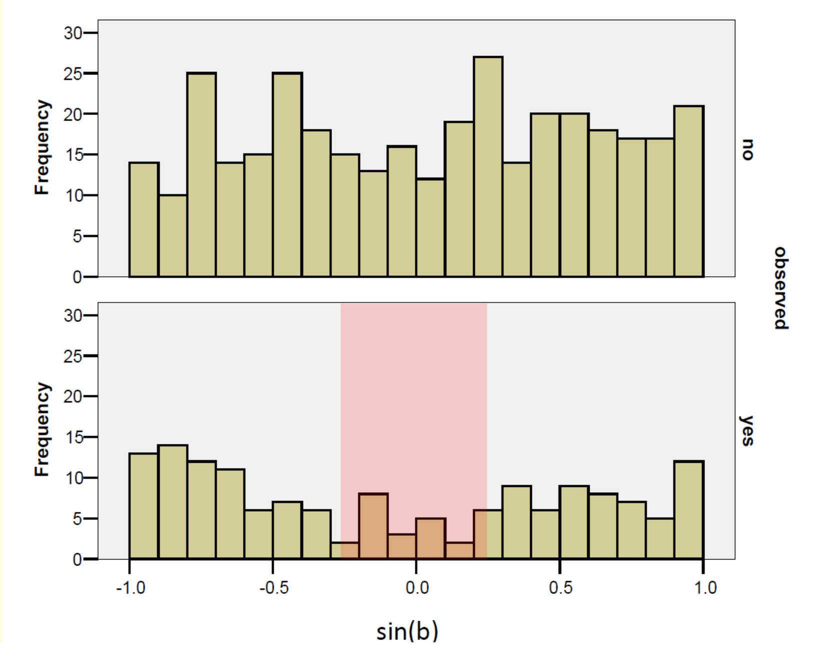

Fig. 1. Distribution of the GRB positions according to the Galactic latitude. Note the depression in the distribution close at low latitudes due to the foreground extinction of the Galactic dust. Pink color marks the excluded area.

In the case if no afterglow was observed a lower bound in the stellar magnitude (upper bound for the observed brightness) was obtained. Seemingly, at low latitudes the depression is not present in the distribution of the cases where only a lower V magnitude bound is determined.

Cox regression

Before we make the Cox regression running we have computed the bivariate correlations between the variables included in the analysis. In this procedure we have taken into account only those cases where both variable used for computing the correlation had measured values. To overcome the problem with the outliers we computed the Spearman's rank correlation which is not sensitive to these.

Spearman's correlations

		logT90	logFlu	logPeak	Pind	logXflu	V
logT90	Cor. Coef.	1.000	.646	-.066	.155	.408	-.024
	Sig.	.	.000	.149	.001	.000	.646
llogFlu	Cor. Coef.	.646	1.000	.539	-.148	.471	-.204
	Sig.	.000	.	.000	.001	.000	.000
logPeak	Cor. Coef.	-.066	.539	1.000	-.282	.141	-.289
	Sig.	.149	.000	.	.000	.018	.000
Pind	Cor. Coef.	.155	-.148	-.282	1.000	.049	.004
	Sig.	.001	.001	.000	.	.406	.944
logXflu	Cor. Coef.	.408	.471	.141	.049	1.000	-.109
	Sig.	.000	.000	.018	.406	.	.080
V	Cor. Coef.	-.024	-.204	-.289	.004	-.109	1.000
	Sig.	.646	.000	.000	.944	.080	.

In the table above we listed all cases having measured values in all variables used in the analysis, pairwise. We marked with red color where the *B* coefficients differ significantly from zero. As we are approaching the detection limit of UVOT, however, only a lover magnitude limit is obtained (see Fig. 2 below) in a significant fraction of cases.

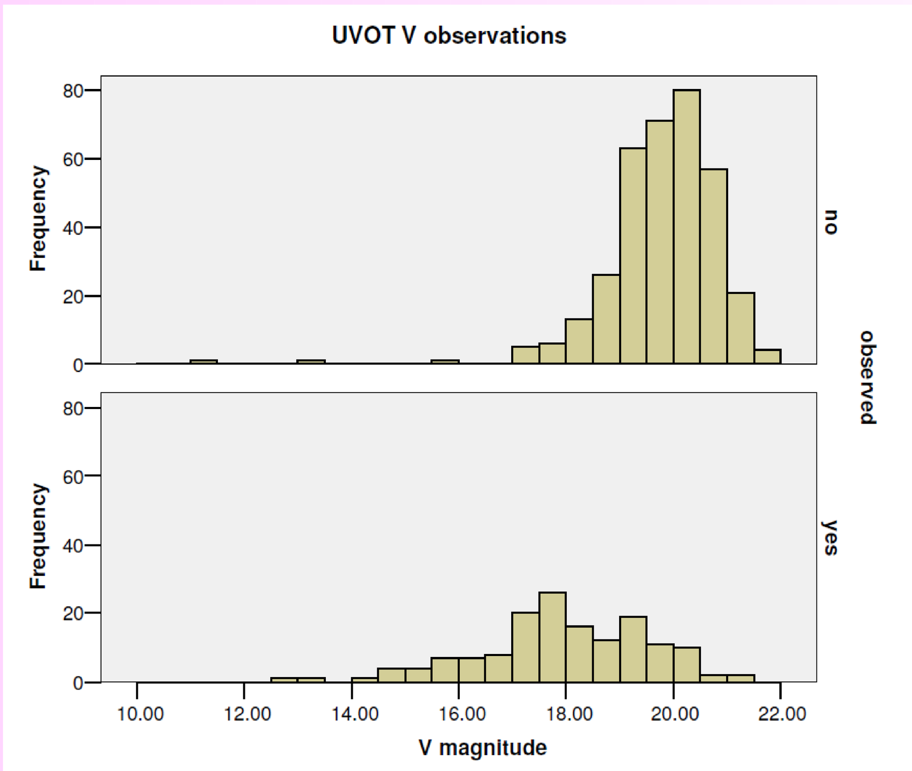

Fig. 2. distribution of the afterglow's brightness in visual magnitude. Note the increasing fraction of censored data as we are approaching the detection limit.

Results of the Cox regression (red color means significant value):

Variables in the Equation

	B	X^2	df	Sig.
logt90	.732	6.346	1	.012
llogflu	-.809	4.099	1	.043
logPeak	1.886	25.438	1	.000
Pind	.058	.055	1	.815
logXflu	-.003	.001	1	.973

Seemingly, the Duration, gamma Fluence and Peak flux has a significant impact on the distribution of the optical brightness, but the Photon index and early X-ray flux do not.

Conclusions

We performed Cox regression in order to look for the impact of the gamma and X-ray properties of the GRBs on the distribution of the afterglow's optical brightness. This approach is necessary since in a significant fraction of cases only an upper bound of the brightness (lower bound in the V magnitude) can be determined. This analysis can extract also the information inherent in the censored (lower bound) data.

The analysis demonstrated that among the *B* coefficients of the covariates in the equation those of the Duration, Fluence and Peak flux significantly differ from zero but it is not the case with the X-ray flux and the gamma Photon index.

The reason for the impact of gamma properties on the optical brightness is probably lying in the energetics of the jet launched from the central engine of the GRB and triggers the afterglow in the surrounding interstellar matter.

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