When gravity meets radiation: the stellar Eddington limit

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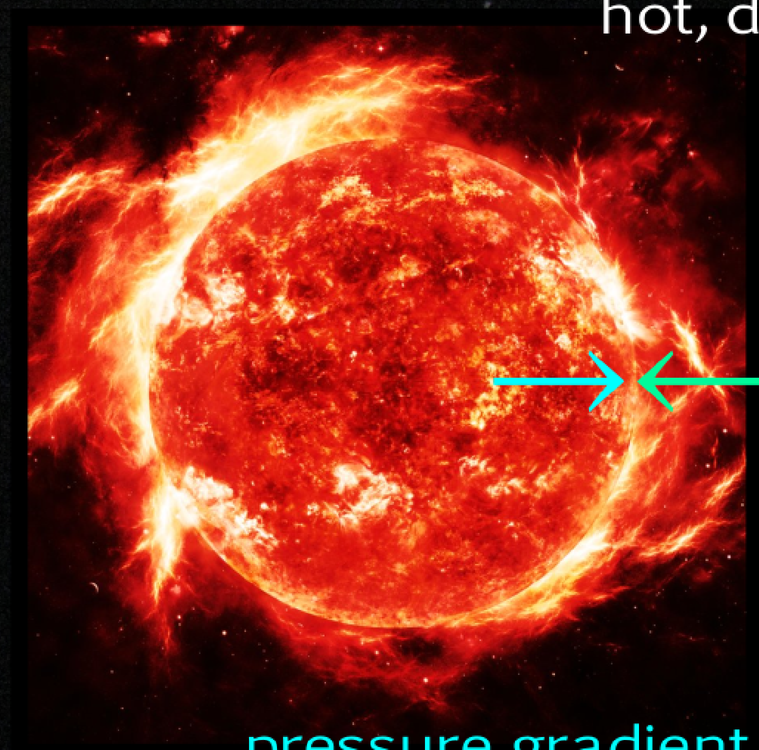
Köln, 27. April 2021

What is a star?



hot, dense plazma

equilibrium: pressure gradient gravity

Theoretical modelling of the stellar structure

$$\frac{\partial r}{\partial m_r} = \frac{1}{4\pi r^2 \rho} \quad \text{equation of definition of mass} \qquad (9)$$

$$\frac{\partial P}{\partial m_r} = -\frac{G m_r}{4\pi r^4} \quad \text{equation of hydrostatic equilibrium} \qquad (10)$$

$$\frac{\partial L_r}{\partial m_r} = \epsilon_{\rm pl} - T\frac{\partial S}{\partial t} \quad \text{equation of energetic balance} \qquad (11)$$

$$\frac{\partial T}{\partial m_r} = -\frac{G m_r T}{4\pi r^4 P}\nabla \quad \text{equation of energy transport,} \qquad (12)$$

Guilera et al. 2011

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composition change due to nuclear burning ?!

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$$\frac{\partial X_i}{\partial t} = \frac{A_i m_u}{\rho}\left(-\Sigma_{j,k} r_{i,j,k} + \Sigma_{k,l} r_{k,l,i}\right) \quad (13)$$

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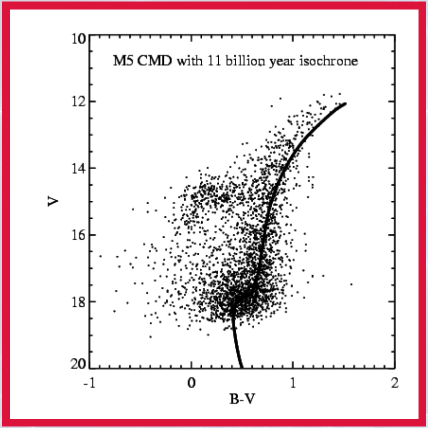
+ Rotation.

Matching theory to observations

Surface properties! → temperature (i.e. colour) X axis
→ luminosity (i.e. brightness) Y axis



Hertzsprung–Russell diagram (HR diagram)

When the equilibrium is compromized:

the Eddington limit

Other reasons for falling out of equilibrium:

- iron core

  → gravitational collapse & SN (due to bounce-back)
- pair-instability

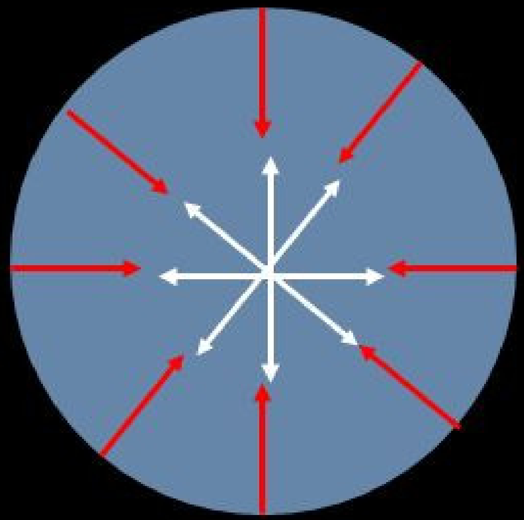
  → grav. collapse & subsequent thermonuclear explosion (PISN) or pulsations (puls-PISN)
- end of a burning phase

  → restructuring, crossing the Herzsprung-gap...
- ...

Eddington limit

Radiative Force

$$g_{rad} = \int_0^\infty d\nu \, \frac{\kappa_\nu F_\nu}{c}$$



Gravitational Force

$$\frac{GM}{r^2}$$

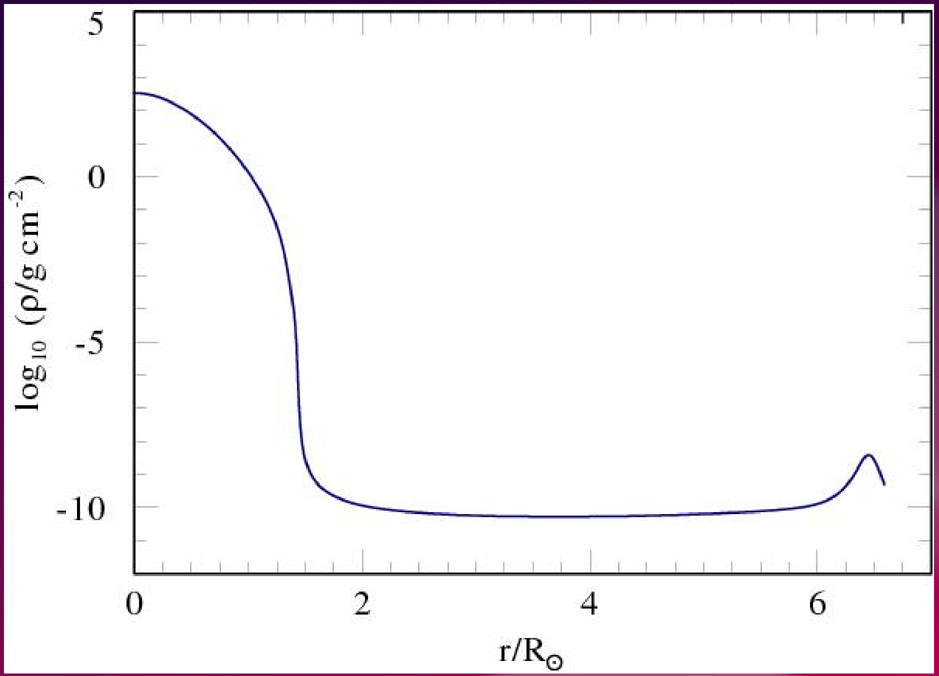
$$\Gamma_e \equiv \frac{g_e}{g} = \frac{\kappa_e L / 4\pi r^2 c}{GM / r^2} = \frac{\kappa_e L}{4\pi G M c}$$

Credit: Stan Owocki

Consequences for the stellar interior

- density (and pressure) inversion *in the envelope*
- no efficient energy transport mechanism here (weak convection)
- → envelope "inflation"
- numerical difficulties...

density inversion:



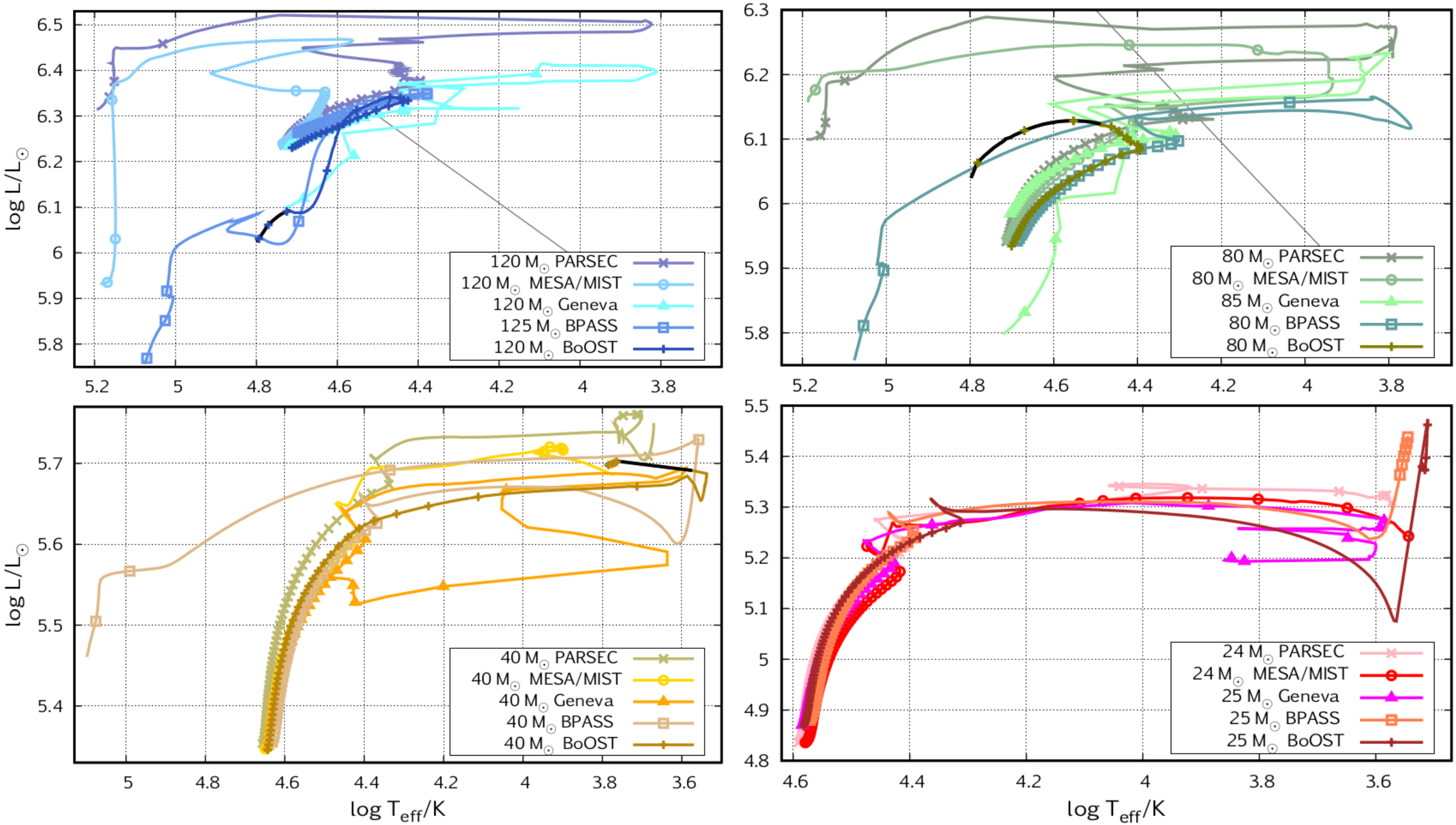
CORE **ENVELOPE**

credit: Götz Gräfener

Is there a solution?

- several “tricks” in the literature
  - various codes use various tricks & methods
  - cf. Szécsi & Agrawal (2021, *submitted*)
- PARSEC (‘Padova’) **artificially limiting the temp. gradient**
- MIST (MESA) **MLT++ formalism *(limiting the superadiabacity*)***
  =changing how convection is treated**
  **difference between the isothermal and adiabatic temperature gradient*
  ***a type of internal mixing*
- ‘Geneva’
- BPASS

  } **artificially enhanced mass loss at the right moment**
- BoOST (‘Bonn’) **inflated envelope & post-processing with ‘normal’ mass loss**



Szécsi & Agrawal (2021, *submitted*)

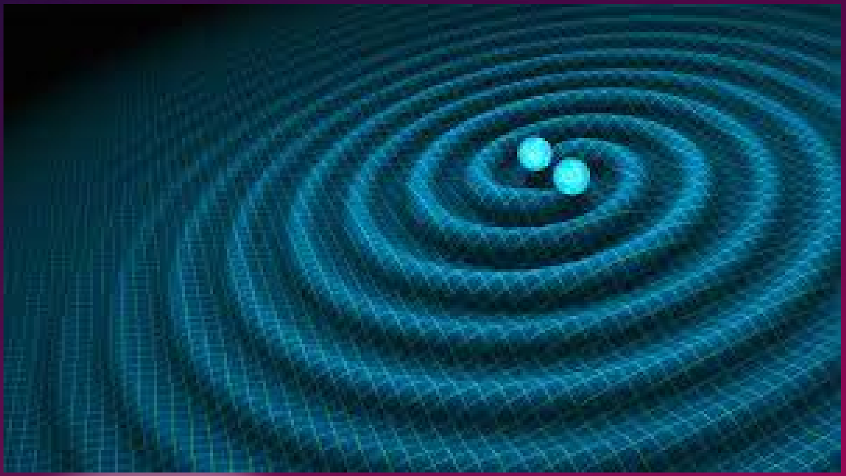
Table 1. Ionizing photon number flux [s^{-1}] in the Lyman continuum emitted *on average* by the stellar models during their lives, cf. Sect. 2.3. The last column provides the amount of Lyman radiation (number of photons [s^{-1}]) that a 10^7 $M_\odot$ population (e.g. a starburst galaxy or a young massive cluster in the Milky Way) containing these massive stars would emit.

M_{ini} [$M_\odot$]	24/25	40	80/85	120/125	pop.
PARSEC	3.7e48	1.3e49	5.5e49	1.0e50	1.08e54
MIST	3.3e48	1.5e49	5.1e49	1.1e50	1.06e54
Geneva	3.5e48	1.2e49	4.6e49	7.8e49	9.27e53
BPASS	3.6e48	1.3e49	4.5e49	7.7e49	9.34e53
BoOST	3.7e48	1.2e49	4.4e49	7.4e49	9.14e53

up to 15% difference!

Szécsi & Agrawal (2021, *submitted*)

Gravitational waves: compact object mergers (e.g. black holes)



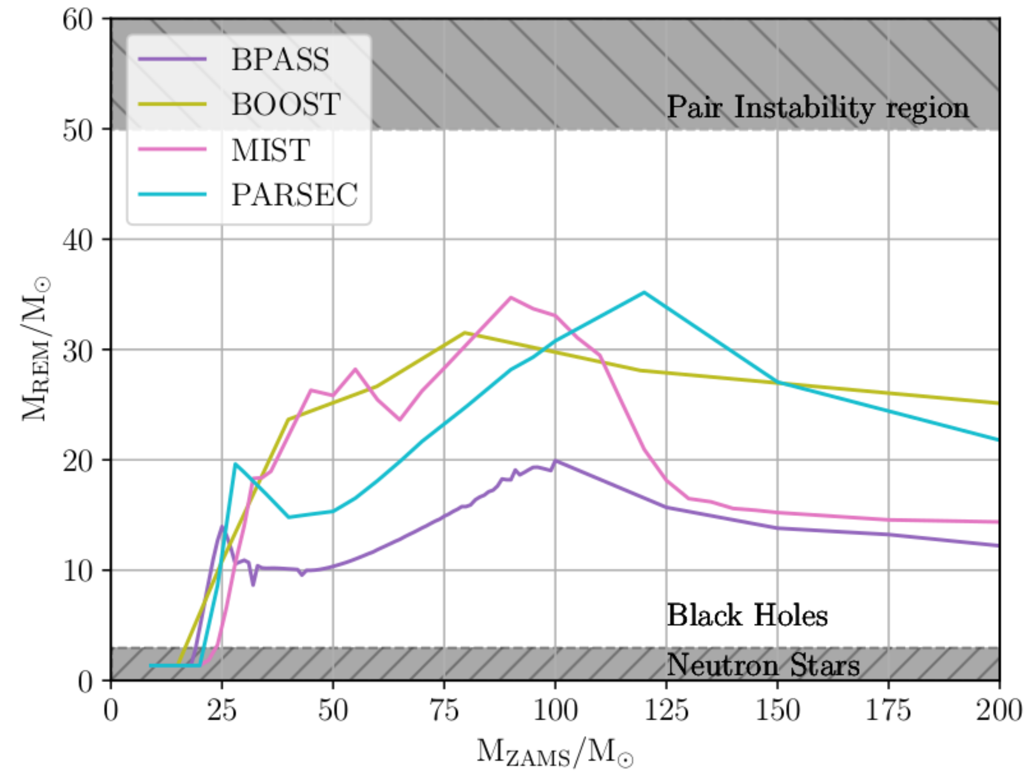


Figure 2. Mass of stellar remnant as a function of the initial mass of the star (near-solar composition). Differences in the assumptions in massive star modelling can cause a variation of up to 20 $M_{\odot}$ in the remnant masses between simulations. Choosing to apply one of these simulations over the others in e.g. gravitational-wave event rate predictions can lead to strikingly different results.

up to 20 $M_{\odot}$ difference!

Szécsi & Agrawal (2021, *submitted*)

Take away messages

- Eddington limit is a thing :)
- stellar evolution above 40 $M_\odot$ has

not reached consensus

- use stellar models with extra caution, be flexible for updates

Thanks!