A Henyey-módszer

Dorottya Szécsi

ELTE

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Louis G. Henyey

- Louis George Henyey (February 3, 1910 – February 18, 1970)
- felesége: Elizabeth Rose Belak (Budapesten szül.), 3 gyerek
- doktori: Yerkes Observatory of the University of Chicago, 1937
- dolgozott: Department of Astronomy at the University of California, Berkeley

Louis G. Henyey

- dolgozott: Department of Astronomy at the University of California, Berkeley
- csillagfejlődési egyenletek automatikus megoldása elektronikusan
- a gravitációs kollapszus és a termonukleáris fúzió közötti átmenet környékének elmélete
- agyvérzés, váratlanul
- kráter a Holdon

Lat: 13.5°N,
Long: 151.6°W,
Diam: 63 km

A probléma

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi\rho r^2}$$

$$\frac{\partial p}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$$

$$\frac{\partial L}{\partial m} = \varepsilon - T\frac{\partial S}{\partial t}$$

$$\frac{\partial T}{\partial m} = \begin{cases} -\dfrac{3}{64\pi^2 ac}\dfrac{\kappa L}{r^4 T^3} & \text{radiatív zóna} \\ \left(\dfrac{\partial T}{\partial m}\right)_{\mathrm{rad}} + \left(\dfrac{\partial T}{\partial m}\right)_{\mathrm{conv}} & \text{konvektív zóna} \end{cases}$$

Az egyenletek

II. THE BASIC DIFFERENTIAL EQUATIONS

The development of the modified form of the computational technique requires that the basic equations be put into a suitable form. Let ξ be a Lagrangian variable and let $m(\xi)$ be the mass inclosed within a sphere designated by ξ, that is,

$$m = m(\xi)\,, \qquad 0 \leq \xi \leq 1\,. \tag{1}$$

Here it is understood that

$$m(0) = 0\,, \quad \text{and} \quad m(1) = M\,, \tag{2}$$

where M is the total mass.

$$\frac{\partial P}{\partial \xi} + \frac{G m \rho}{r^2}\frac{\partial r}{\partial \xi} = 0\,,$$

$$m' - 4\pi r^2 \rho \frac{\partial r}{\partial \xi} = 0\,,$$

$$\frac{\partial l}{\partial \xi} - m'\left[\epsilon - \frac{\partial E}{\partial t} - P\frac{\partial}{\partial t}\left(\frac{1}{\rho}\right)\right] = 0\,;$$

and for radiative-conductive transfer of energy

$$\frac{\partial T}{\partial \xi} + \frac{3\kappa\rho l}{64\pi\sigma T^3 r^2}\frac{\partial r}{\partial \xi} = 0\,, \tag{6}$$

(where κ, the opacity, includes the effect of electron conduction) or for convection

$$\frac{\partial E}{\partial \xi} + P\frac{\partial}{\partial \xi}\left(\frac{1}{\rho}\right) = 0\,. \tag{7}$$

The symbol m' represents the ordinary derivative of $m(\xi)$ with respect to ξ. As usual ρ represents the density, P the pressure, T the temperature, l the luminosity, and r the radius at any interface within the star. E is the internal energy per unit mass and ϵ the thermonuclear energy release per unit mass and time. M, R, and L are the mass, radius, and luminosity of the whole star.

Diszkretizálás I.

- diszkrét tömegpontok pozíció szerint rendezve: $\xi_j (j = 0, 1, \ldots, J)$, $\xi_0 = 0$ középpont, $\xi_J = 1$ felszín
- jelölés: T_j, q_j stb.
- megfelelően kis változás két pont között
- két kategória: állandó pontok, időszakos pontok (az iteráció során beteszik, ha kell)
- differencia-egyenletek ún. „középpontozott” alakban:

Diszkretizálás II.

$$p_{j+1} - p_j + \frac{G m_{j+1/2} (q_{j+1} + q_j)^3 (r_{j+1} - r_j)}{(p_{j+1} + p_j)^3 (r_{j+1} + r_j)^2} = 0 .$$

$$\frac{8}{\pi} m_{j+1/2}' (\xi_{j+1} - \xi_j) - (q_{j+1} + q_j)^3 (r_{j+1} + r_j)^2 (r_{j+1} - r_j) = 0 .$$

$$F_{j+1} (\xi_{j+1} + \xi_j)(3\xi_{j+1} - \xi_j) + F_j (\xi_{j+1} + \xi_j)(\xi_{j+1} - 3\xi_j)$$
$$- 2 m_{j+1/2}' (\xi_{j+1} - \xi_j) \left[2 (\epsilon_{j+1} \epsilon_j)^{1/2} - \frac{E_{j+1} + E_j - E_{j+1}{}^n - E_j{}^n}{\Delta t} \right.$$
$$\left. + 3 \left(\frac{p_{j+1} + p_j}{q_{j+1} + q_j} \right)^4 \frac{q_{j+1} + q_j - q_{j+1}{}^n - q_j{}^n}{\Delta t} \right] = 0 .$$

$$T_{j+1} - T_j - \frac{(K_{j+1} + K_j)(\xi_{j+1} + \xi_j)^2 (F_{j+1} + F_j)(p_{j+1} - p_j)}{m_{j+1/2}} = 0 .$$

$$E_{j+1} - E_j - 3 \left(\frac{p_{j+1} + p_j}{q_{j+1} + q_j} \right)^4 (q_{j+1} - q_j) = 0 .$$

Határfeltételek

$$r_0 = 0$$

$$l_0 = 0\,.$$

- $l = \xi^2 F$ pszeudofluxus
- felszínen: egy másik program iteratívan variálva határozza meg

$$r_{J-1} = f_1(R, L)\,,$$

$$F_{J-1} = f_2(R, L)\,,$$

$$T_{J-1} = f_3(R, L)\,,$$

$$q_{J-1} = f_4(R, L)\,.$$

A gép

- UNIVAC = UNIVersal Automatic Computer (Livermore Radiation Laboratory)
- 36 Williams tubes with a capacity of 1024 bits each
- 1 Williams tube was five inches in diameter

Megoldás I.

IV. SOLUTION OF THE DIFFERENCE EQUATIONS

In order to obtain a description of the solution of the difference equations in a concise form, we adopt a vector notation.

The first two difference equations (22) and (23) may be written in vector notation as

$$\phi_{j+1/2}\,(u_j,\,v_j;\,u_{j+1},\,v_{j+1}) = 0\,, \tag{34}$$

while the third and fourth equations, (24) and (25) or (26), for radiative or convective equilibrium, respectively, are written

$$\psi_{j+1/2}\,(u_j,\,v_j;\,u_{j+1},\,v_{j+1}) = 0\,. \tag{35}$$

Thus $\phi_{j+1/2}$ and $\psi_{j+1/2}$ are two-component vectors, each a function of the four vectors u_j, v_j, u_{j+1}, v_{j+1}, where

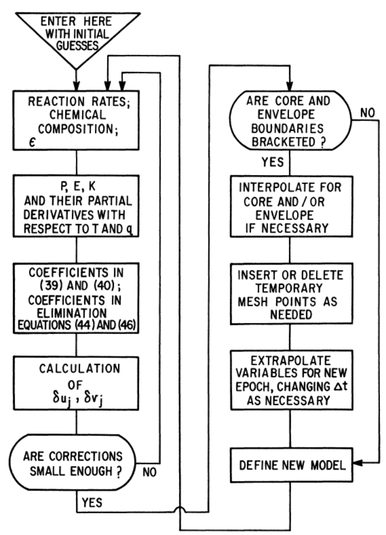
$$u_j = \begin{pmatrix} r_j \\ F_j \end{pmatrix} \quad \text{and} \quad v_j = \begin{pmatrix} T_j \\ q_j \end{pmatrix}.$$

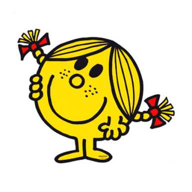
a vektorok variálásával linearizált egyenletrendszer:

$$\phi_{j+1/2} + \delta u_j \frac{\partial}{\partial u_j} \phi_{j+1/2} + \delta v_j \frac{\partial}{\partial v_j} \phi_{j+1/2} + \delta u_{j+1} \frac{\partial}{\partial u_{j+1}} \phi_{j+1/2} + \delta v_{j+1} \frac{\partial}{\partial v_{j+1}} \phi_{j+1/2} = 0$$

$$\psi_{j+1/2} + \delta u_j \frac{\partial}{\partial u_j} \psi_{j+1/2} + \delta v_j \frac{\partial}{\partial v_j} \psi_{j+1/2} + \delta u_{j+1} \frac{\partial}{\partial u_{j+1}} \psi_{j+1/2} + \delta v_{j+1} \frac{\partial}{\partial v_{j+1}} \psi_{j+1/2} = 0\,.$$

Megoldás II.


ENTER HERE WITH INITIAL GUESSES
REACTION RATES; CHEMICAL COMPOSITION; ε
P, E, K AND THEIR PARTIAL DERIVATIVES WITH RESPECT TO T AND q
COEFFICIENTS IN (39) AND (40); COEFFICIENTS IN ELIMINATION EQUATIONS (44) AND (46)
CALCULATION OF δuj, δvj
ARE CORRECTIONS SMALL ENOUGH ?
NO
YES
ARE CORE AND ENVELOPE BOUNDARIES BRACKETED ?
NO
YES
INTERPOLATE FOR CORE AND / OR ENVELOPE IF NECESSARY
INSERT OR DELETE TEMPORARY MESH POINTS AS NEEDED
EXTRAPOLATE VARIABLES FOR NEW EPOCH, CHANGING Δt AS NECESSARY
DEFINE NEW MODEL



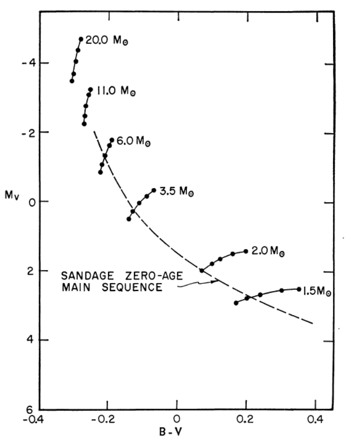
Összevetés a megfigyelésekkel

Szimuláció

- pontok: 25% H-csökkenés
- számolás kezdete: homogén anyag, konvektív magban pp és CNO ciklus
  - X=68%, Y=31%, Z=1%
- vége: elfogy a H (héjban égés beindul ezután)

Megfigyelés

- ZAMS: Sandage, 1957
- 10 nyílthalmazra illesztéssel
- elég nagy hibája van...



Források

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- Henyey, L. G.; Lelevier, Robert; Levee, R. D. – Evolution of Main-Sequence Stars (Astrophysical Journal, vol. 129, p.2)
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- http://the-moon.wikispaces.com/Henyey

Köszönöm a figyelmet!