Astrophysics Seminar
Introduction to Cosmology
Part 2.

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Summary of the Previous Lecture I.

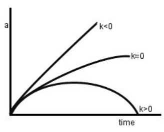
- Cosmological Principle: $> 100\ \mathrm{Mpc}$ Universe is homogeneous and isotropic
- Hydrodynamical approximation:
  - Continuity equation
  - Euler equation
  - Laplace–Poisson equation: $\partial_{ii} U = 4\pi G\rho$
  - Universe cannot be static $\rightarrow \partial_{ii} U + \lambda = 4\pi G\rho$ (Neumann and Seeling)
  - Friedman equation: $\frac{1}{2}\dot{R}^2 - \frac{MG}{R} = E$ (energy conservation)
- Scale parameter: $a(t) = \frac{R(t)}{R_0}$
- General relativity – Einstein equation, with Schwartzschild metric:

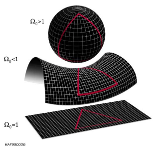
$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{8\pi G}{3}\rho = -\frac{k}{a^2}, \qquad k = -1, 0, 1$$

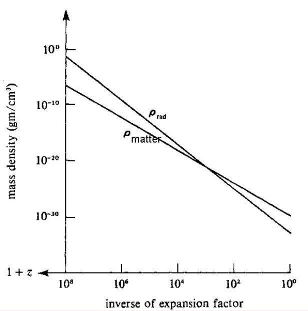
($k = 0$ special case: Friedman–Robertson–Walker metric)

Summary of the Previous Lecture II.

- Resolving the Friedman equation in the cases $k = 1$ closed, $k = 0$ flat, $k = -1$ open Universe:







- Cosmical era dominated by matter or radiation:
  - If the space is dominated by non-relativistic moving particles:
    $$\rho_m \sim \frac{1}{a^3},\ a(t) \sim t^{2/3}$$
  - If the space is dominated by relativistic particles (photons, maybe neutrinos):
    $$\rho_{rad} \sim \frac{1}{a^4},\ a(t) \sim t^{1/2}$$

Cosmological Constant: Λ, Ω_Λ

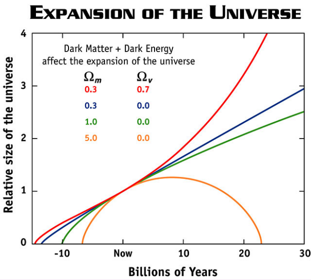
- Observation: expansion of Universe is accelerating
- Laplace–Poisson equation + λ additive contribution: cosmological constant, Λ
- Change of Friedmann equation: $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3}$
- After integrating and transforming: $\frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 - \frac{4\pi G}{3}\left(\rho_m + \rho_{sug} + \rho_\Lambda\right) = -\frac{k}{2a^2}$, where $\rho_\Lambda = \frac{\Lambda}{8\pi G}$
- Emphasizing $H^2(t)$: $\frac{H^2}{2}\left(1 - \Omega_m - \Omega_{rad} - \Omega_\Lambda\right) = -\frac{k}{2a^2}$, where $\Omega_\Lambda = \frac{\lambda}{3H^2}$
- We take: $1 - \Omega_m - \Omega_{rad} - \Omega_\Lambda = \frac{-k}{H^2 a^2} = \Omega_k$
- Ω_k: contribution of space curvature to the energy density

Evolving Universe I.

  - 20th century: $\Omega_\Lambda = 0$, $\Omega_k = 0$ but measurements prove: $\Omega_\Lambda \neq 0$
- Hubble constant is "constant" for all galaxies in a timepoint, but changes in time, because ρ changes with a: $\rho_m \sim a^{-3}$, $\rho_{sug} \sim a^{-4}$, ρ_Λ is independent from $a \Longrightarrow$

$$H^2(a) = H_0^2[\Omega_{\Lambda,0} + \Omega_{m,0}a^{-3} + \Omega_{sug,0}a^{-4}]$$

  - From measurements: $\Omega_{m,0} \approx 0.3$, $\Omega_{\Lambda,0} \approx 0.7$, $\Omega_{sug,0} \approx 0$. We get the past and future state of the Universe:



Evolving Universe II.

- Now we seek the possible Universes depending from Ω_Λ and Ω_m. Let $\Omega_k \neq 0$:

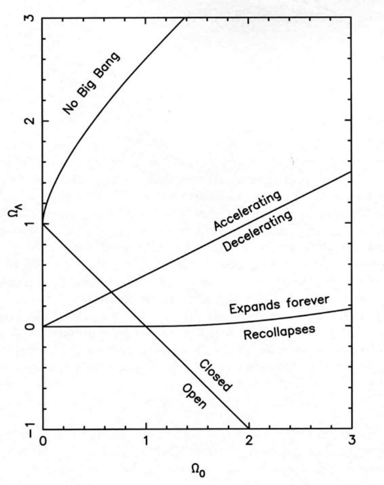
$$H^2(a) = H_0^2[\Omega_{\Lambda,0} + \Omega_{m,0}a^{-3} + \Omega_{sug,0}a^{-4} + \Omega_{k,0}a^{-2}]$$

- We neglect the radiation component and replace $1 - \Omega_m - \Omega_\Lambda = \Omega_k$:

$$\frac{H^2(a)}{H_0^2} = [\Omega_{\Lambda,0}(1 - a^{-2}) + \Omega_{m,0}(a^{-3} - a^{-2}) + a^{-2}]$$

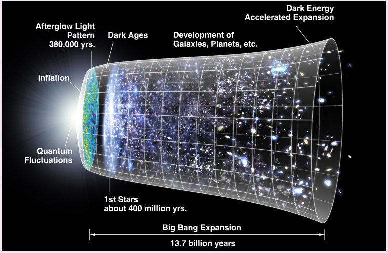
- It is easy to resolve numerically. The resolution on the Ω_m–Ω_Λ-plane:

Evolving Universe III.



- $\Omega_k = 0 \rightarrow \Omega_m + \Omega_\Lambda = 1$: spacelike flat Universe
- Both cases contain the possibility of Recollapsing and Forever Expanding Universe
- If Ω_Λ is big enough: its repulsive effect withholds the existence of $a = 0$ in the past
- $a(t)$ series expansion: the parabolic member's factor is the *decelerating parameter*, q_0.
  - In our plane, $q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda$.
  - With $q_0 = 0$, we get the $\Omega_\Lambda = \frac{1}{2}\Omega_m$ line
  - above: accelerating expansion, below: decelerating expansion (if it does not contract)
- Our Universe is accelerating now!

Evolving Universe IV.



- When the turnout from the decelerating to the accelerating expansion happened: $q = 0$
  - $\rightarrow \Omega_\Lambda = \frac{1}{2}\Omega_m$
  - we know that $\Omega_{\Lambda,0} = \frac{1}{2}\Omega_{m,0} a^{-3}$
  - and $\Omega_{\Lambda,0} = 0.7$, $\Omega_{m,0} = 0.3$ now so we get
- $a \approx 0.6$, $\Omega_m \approx 0.67$, $\Omega_\Lambda \approx 0.33$

Evidences of the Theory I.

- Hubble's law (1929):
  - Distant galaxies spectra: lines shift to the red range
  - Redshift: $z = \dfrac{\lambda - \lambda_0}{\lambda} \approx \dfrac{v}{c}$
  - Many galaxies have been measured: Hubble diagram, $v = Hd$
  - Cosmological meaning of redshift: $1 + z = \dfrac{a_0}{a(t)}$

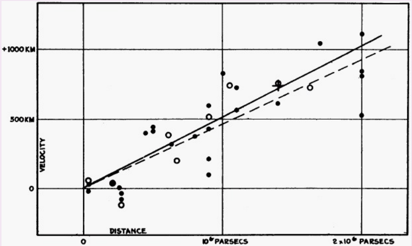


FIGURE 1

Velocity-Distance Relation among Extra-Galactic Nebulae.

Evidences of the Theory II.

- CBM (1965):
  - Once in great tempearture ($> 3000\ \mathrm{K}$), H-atom was not stable, fell apart. Photons could scatter on free electrons and protons.
  - It ended below $3000\ \mathrm{K}$. Protons and electrons *recombined* to H-atoms, matter became transparent to photons, so they departed from here and we see them now as Cosmic Microwave Background.
  - Universe is 1000 times bigger now: photons temperature has to be $\sim 3\ \mathrm{K}$.
- Quantity of light elements:
  - There was a time, when nucleus was not stable. Then protons and neutrons combined to H, D, He, Li.
  - But here the process stopped: free path length of the the elements became larger than the distance of the horizon (temperature and pressure decreased).
  - Now we see 75% H and 25% He in the Universe (form astronomical observations). It is precisely the value of the calculation.

Problems I.

- Problem of the horizon
  - Universe is expanding with $a(t) \sim t^{\frac{2}{3}}$.
  - $R_{now} = ct_{now}$, $R_{then} = 0.001 R_{now}$.

$$\frac{R_{then}}{ct_{then}} = \left(\frac{R_{then}}{R_{now}}\right)\left(\frac{R_{now}}{ct_{now}}\right)\left(\frac{ct_{now}}{ct_{then}}\right) = \sqrt{\frac{R_{now}}{R_{then}}} \approx 30$$

  - Photons incoming from the opposite site of the horizon could not be in causal relation.
  - CMB photons have the similar $2.73\ \mathrm{K}$ temperature. How could they thermalize without changing information?

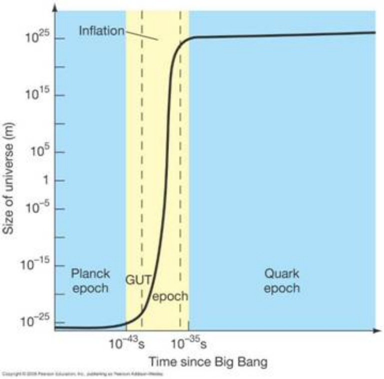
Problems II.

- Problem of metric being flat
  - $1 - \Omega_{total} = \Omega_k = \frac{-k}{H^2 a^2} \approx \frac{1}{\dot{a}^2} \approx t^{\frac{2}{3}} \rightarrow \Omega_{total}$ is going far from 1 with time.
  - We have bounds: $0.5 < \Omega_{total} < 2.0$ (there is enough matter to evolve structures but not enough to begin to contract)
  - In the beginning, Ω_{total} must have been 1 with 10^{-30} precision! Why is the metric so precisely flat?
- Problem of conformation of structures
  - On the scale $< 100\ \mathrm{Mpc}$, the matter is inhomogenous (stars, galaxies, clusters). We see inhomogenity in the CMB of the order of 10^{-5}. What caused that?

Inflation I.

- A. Guth, 1981: before the beginning, there was a primal matter, called *inflaton*, whose density changed slowly (like the cosmological constant). It effected a rapid expansion, where $a(t)$ grew with the order of 30, the second member is irrelevant:
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \approx \frac{8\pi G}{3}\rho_{infl} \equiv H_{infl}^2$$
- Resolving the equation: $a_{out} = a_{in} e^{H_{infl}(t_{out} - t_{in})}$
- In that era, the characteristic scale was the Plack scale, so $a_{in} = R_{Planck}/R_{now} \approx 10^{-61}$, $t_{be} = 10^{-33}\ \mathrm{s}$
- Universe expanded exponentially and cooled adiabaticly.
- Then, the *inflaton* decayed, the Universe warmed up again, and began to evolve by the Friedman equation.

Inflation II.

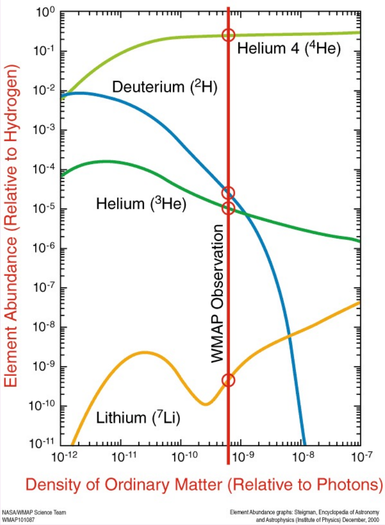


- Inflation solves the problems:
  - Before it, everything was in causal connection. During the inflation, things went out over the horizon, now they come back.
  - Metric could be whatever, after the inflation it is irrelevant, so the Universe can be flat now.
  - Before the inflation, the scale was microscopic and there were quantum fluctuations in the energy density. During the $10^{-31}\ \mathrm{s}$ of the inflation, they grew to macroscopic scale.
- We do not know what happened *before* the inflation. (”Quantum gravitation theory”)

Thermal History of the Universe I.

- Planck time: 10^{-43} s. Before it, we cannot be sure in anything: $\Delta E \Delta t \simeq \hbar$
- GUT (Great Unified Theory): $t = 10^{-37}$ s, 10^{19} GeV, electroweak and strong interactions are unified, barion-antibarion asimetry
- 10^2 GeV: electroweak interaction divide to electromagnetic and weak interactions. W-Z bosons, quarks, leptons get mass.
- Quarks compose mesons, barions: $t = 10^{-5}$ s, 200 MeV
  - Annihilation of charged particles: pions, muons, electrons
  - Neutrinos had scattered on muons and electrons, now they decouple from the matter ($3 \cdot 10^{10}$ K). It heats the Universe.
  - Neutrino Background has to be $\sim 1.9K$, like CMB. (We see just thermonuclear neutrinos from starcores.)

Thermal History of the Universe II.



NASA/WMAP Science Team
WMAP101087

Element Abundance graphs: Steigman, Encyclopedia of Astronomy and Astrophysics (Institute of Physics) December, 2000

- Primordial Nucleosynthesis: 1 MeV
  - Light particles arose
  - From calculations, 25% of the barionic matter is He. If He came into existence just in stars, 6 times lesser He would be now.
- Recombination: 3000 K, $t = 380000\ \mathrm{yrs}$, photons decouple from protons and electrons composing H atom. CMB.
- First light: stars begin to shine

Precision Cosmology

- Standard Model with Inflation became accepted
- Parameters have to be mesured precisely: Hubble constant, Ω_m, $\Omega\Lambda$
- Methods from 1990 to today:
  - Analysing the spectra of CMB fluctuation
  - Measuring distance of the Ia type supernovae
  - Mapping the great scale structure of the Universe

Sources

- Frei-Patkós: Inflációs kozmológia (Typotex, 2005)
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- http://www.kcvs.ca/martin/astro/kingsu/unit6/151/chp18_files/cosmicabund.jpg
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Thank You for your Attention!