

Gravitational-wave progenitors

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Lecture #10

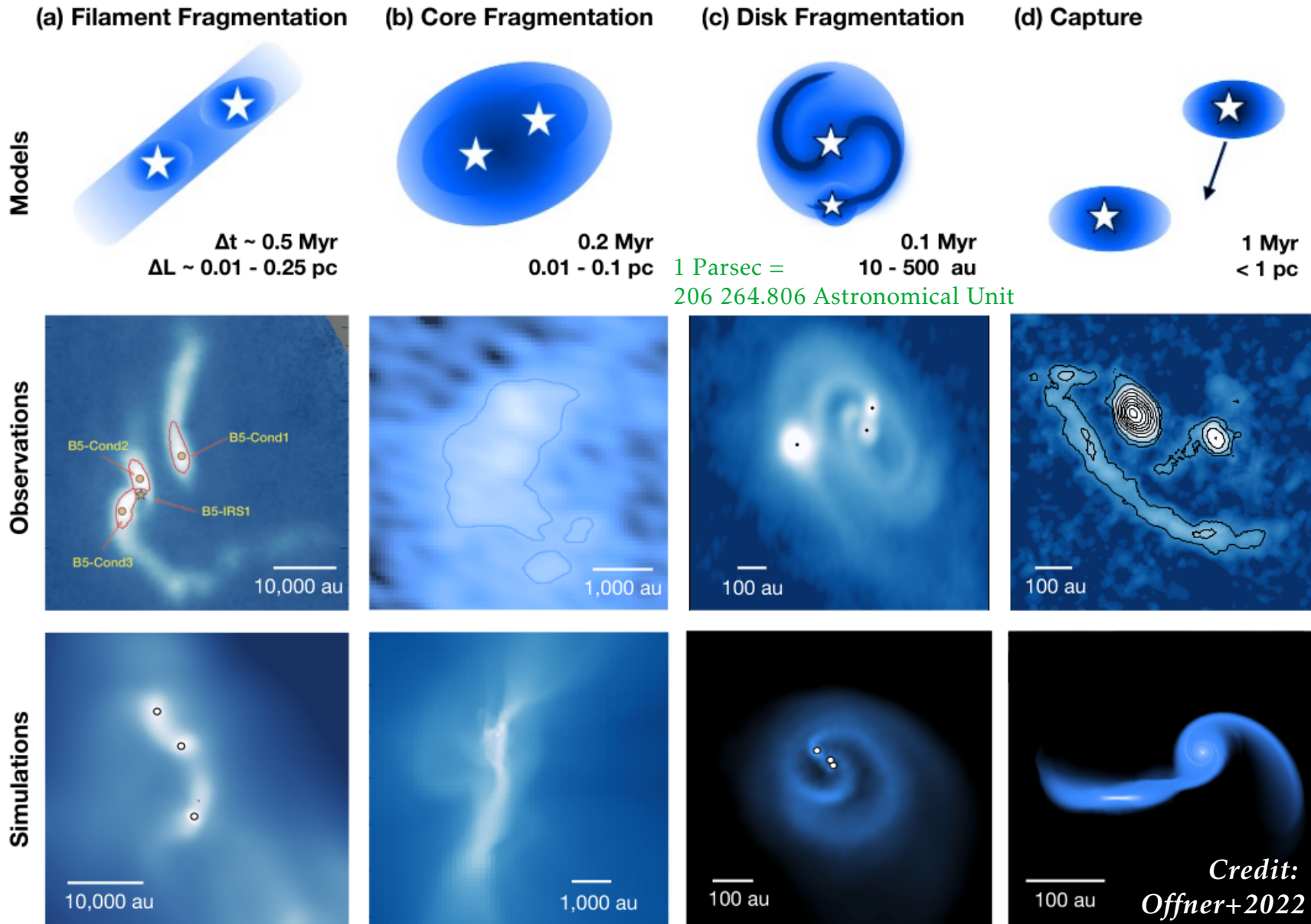
NCU, Summer Semester 2022

*Previously
on GW-progenitors...*

Stars sometimes form in

multiples

binaries, triples, quadruples...



*Credit:
Offner+2022*

Our strategy is/has been:

start with
Massive Stars at Solar Z

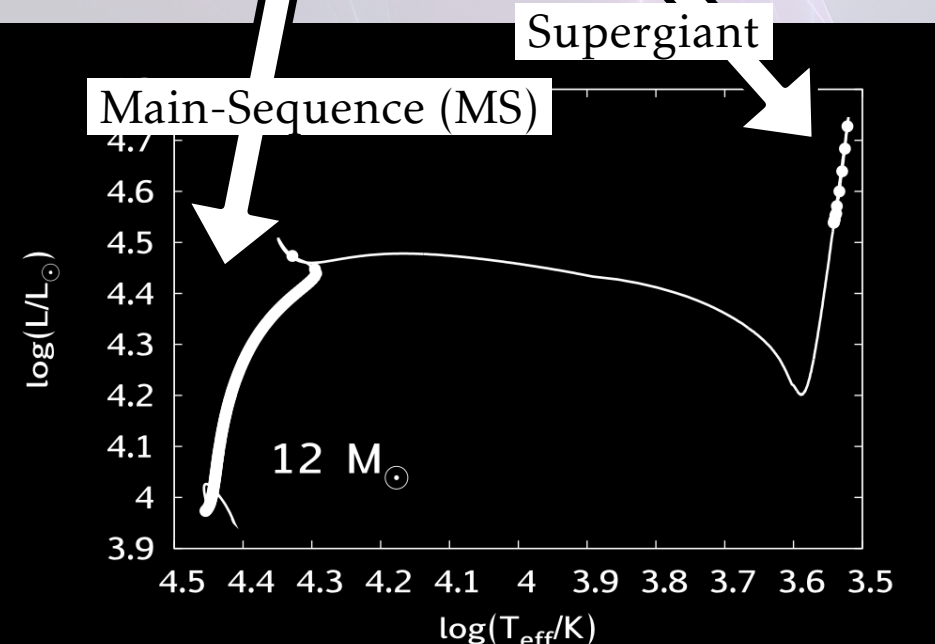
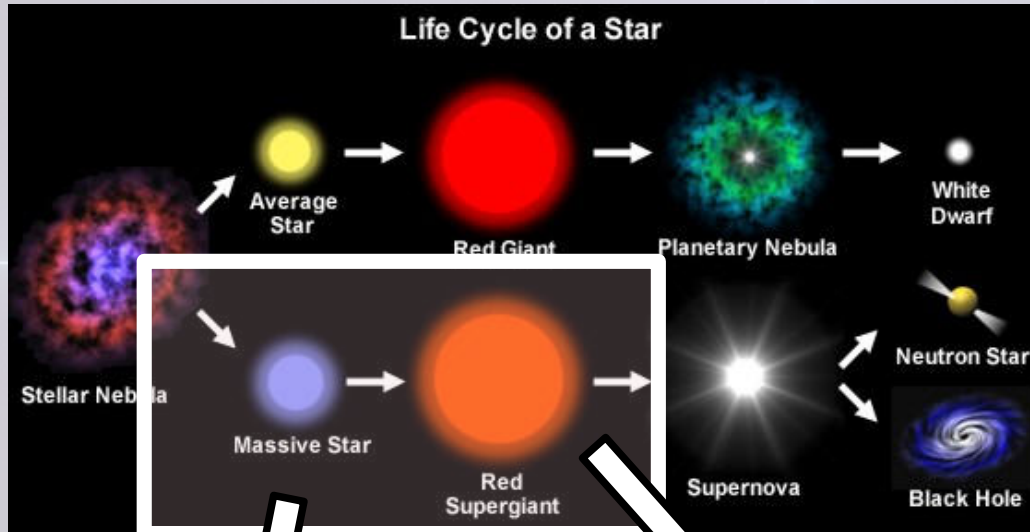
- sub-Solar metallicities?
- fast-rotating stars?
- stars in a binary system?

with Solar Z, no rotation

Imagine two (massive) stars!

One (massive) star alone:

Two of them next to each other:

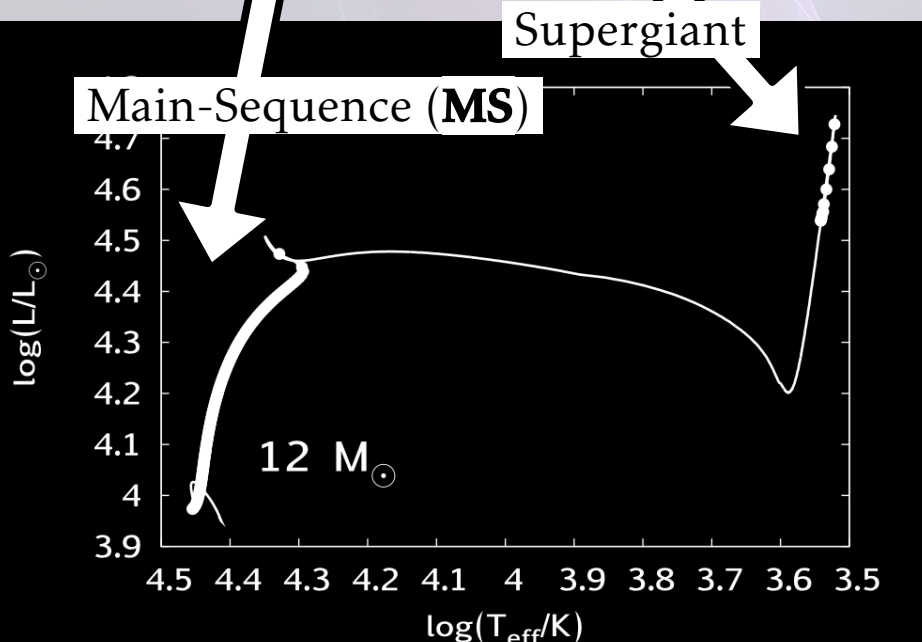
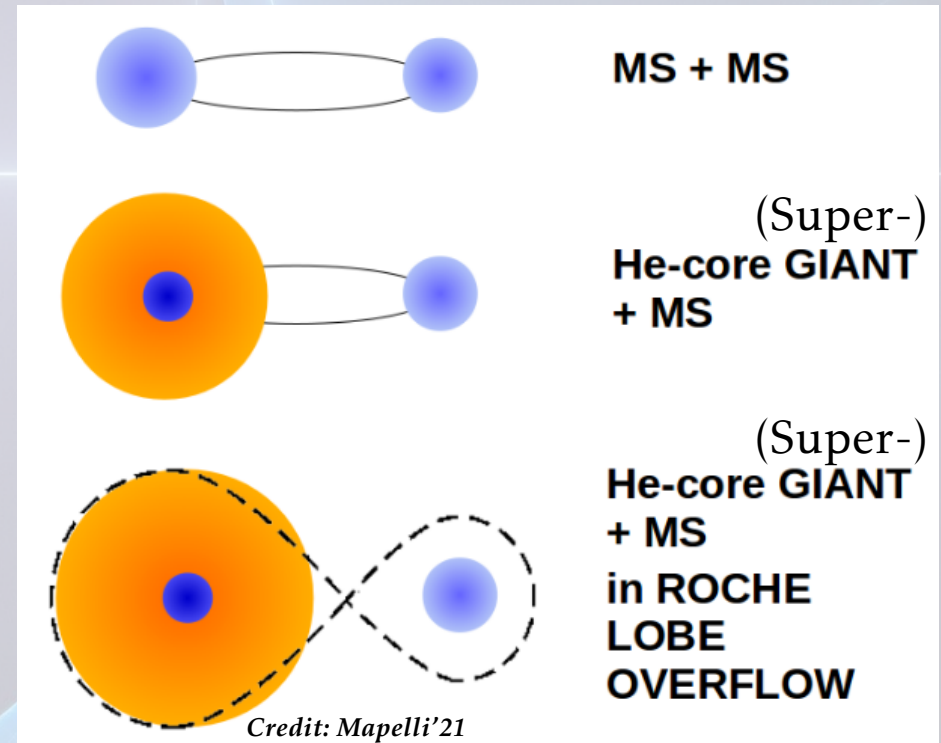
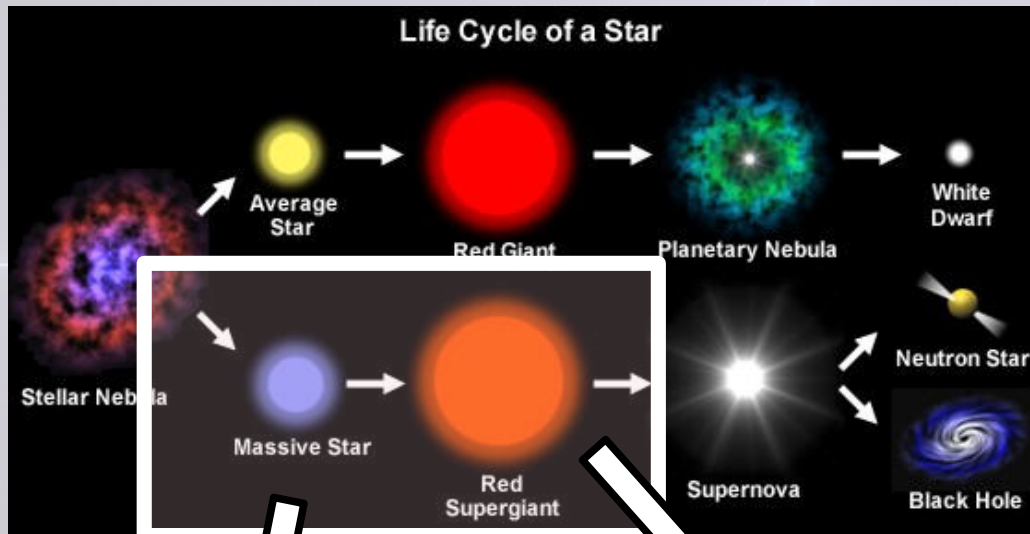


with Solar Z, no rotation

Imagine two (massive) stars!

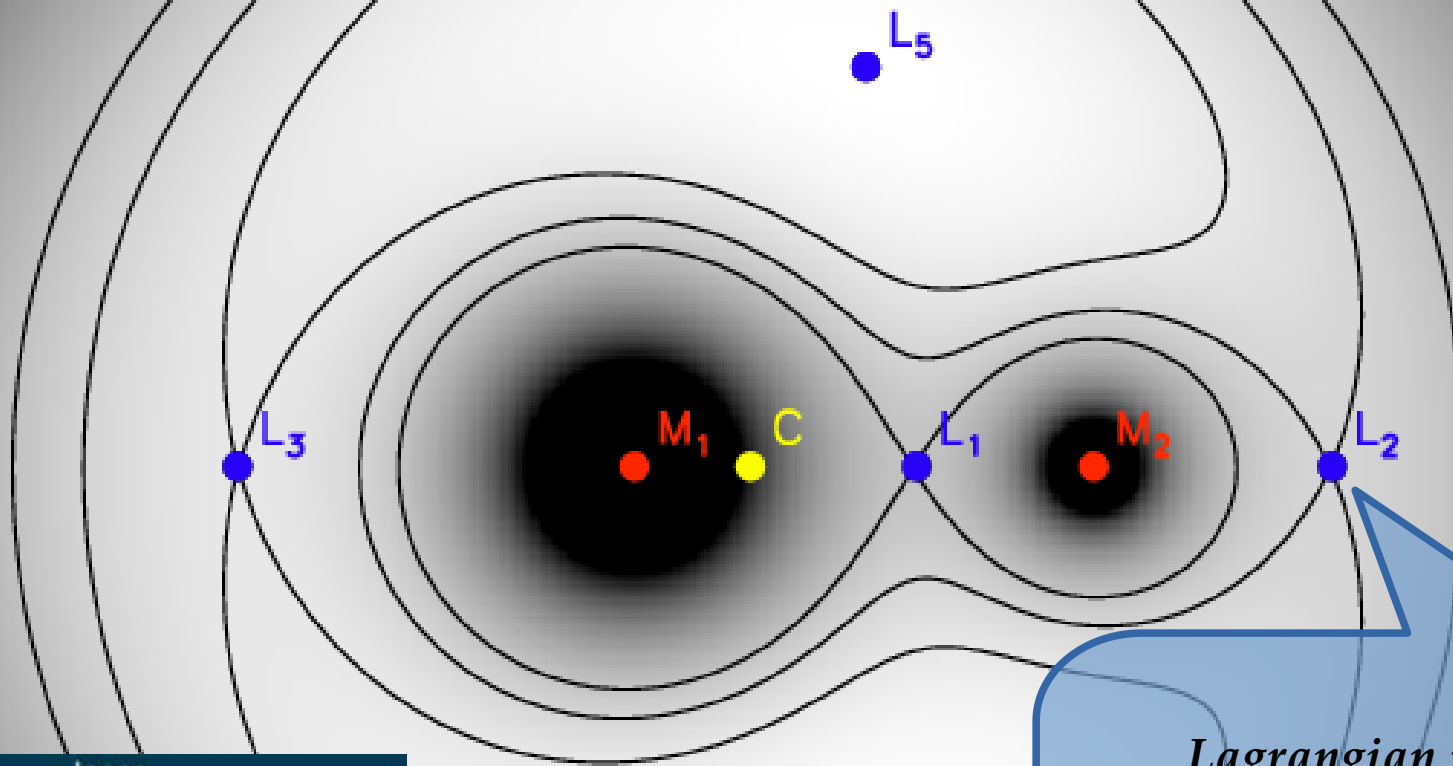
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Two of them next to each other:

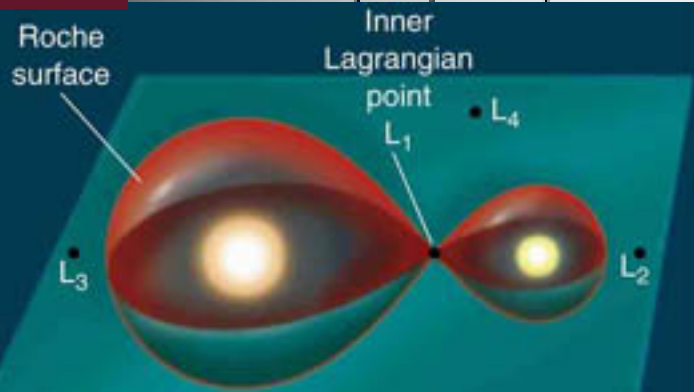


- $\tau(m) \sim m^{-2.5}$
 - Sun's lifetime: $\sim 10^{10}$ yrs
 - an $8 M_{\odot}$ star's lifetime: $\sim 5 \times 10^7$ yrs
 - a $100 M_{\odot}$ star's lifetime: $\sim 2 \times 10^6$ yrs

The most important concept: Roche-lobe



in 3D:

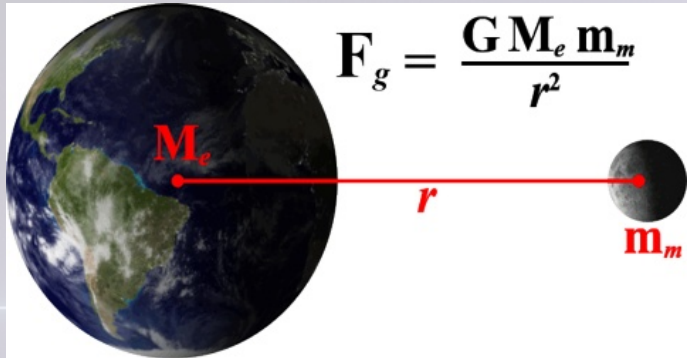


Credit: D.Darling

Lagrangian points:
where the gravitational
forces of the two
bodies and the centrifugal
force balance
each other

Credit: Bonneau+15

Gravitational equipotential surfaces



$$F_g = \frac{GM_e m_m}{r^2}$$

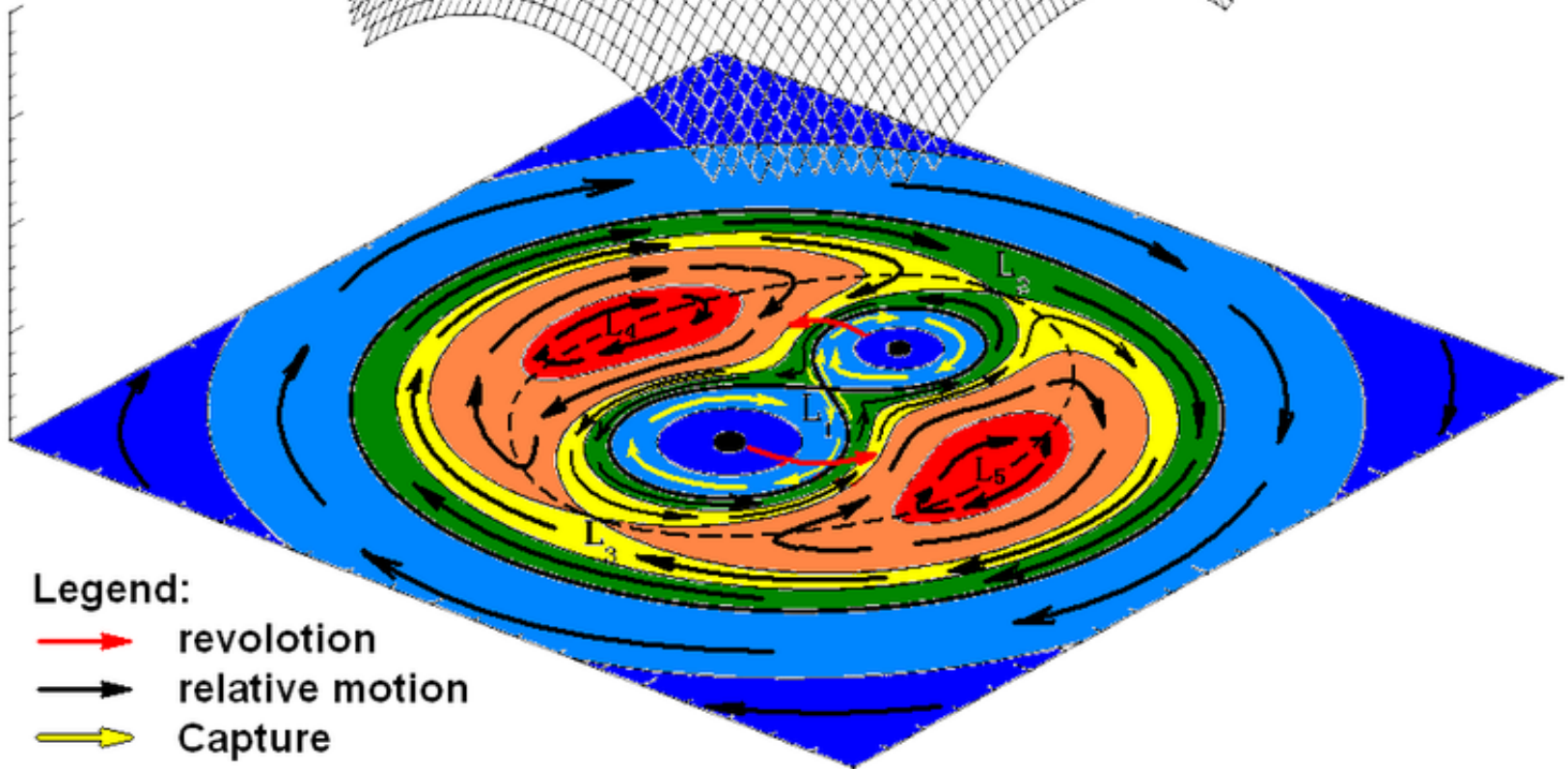
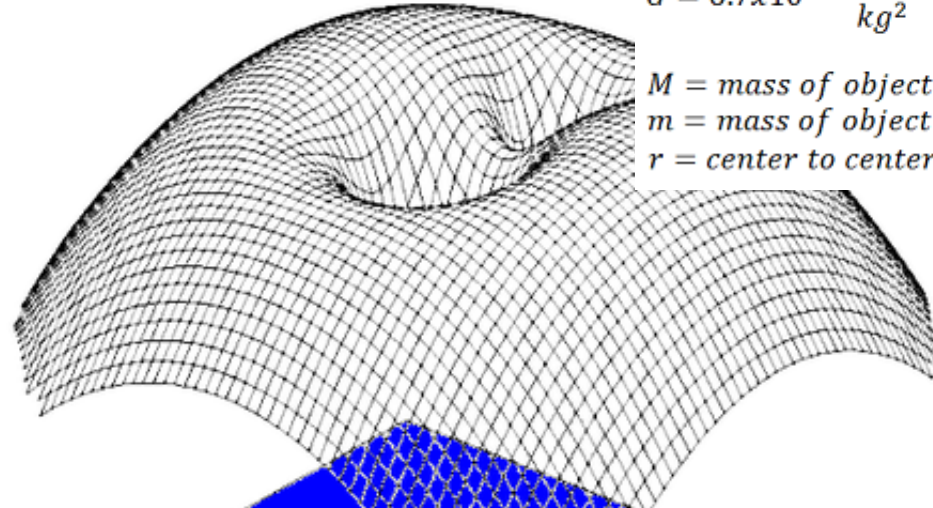
$$G = 6.7 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

$$U_g = \frac{-GMm}{r}$$

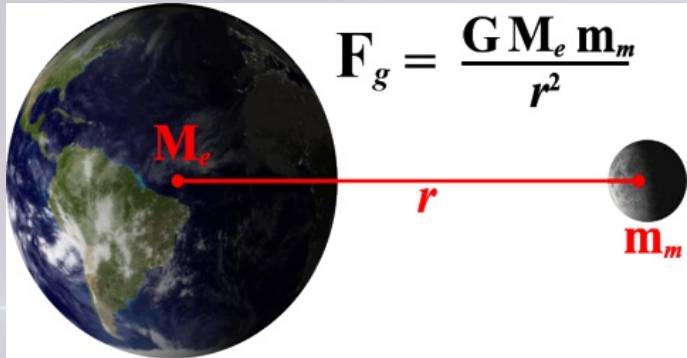
M = mass of object 1

m = mass of object 2

r = center to center distance between M and m



Gravitational equipotential surfaces



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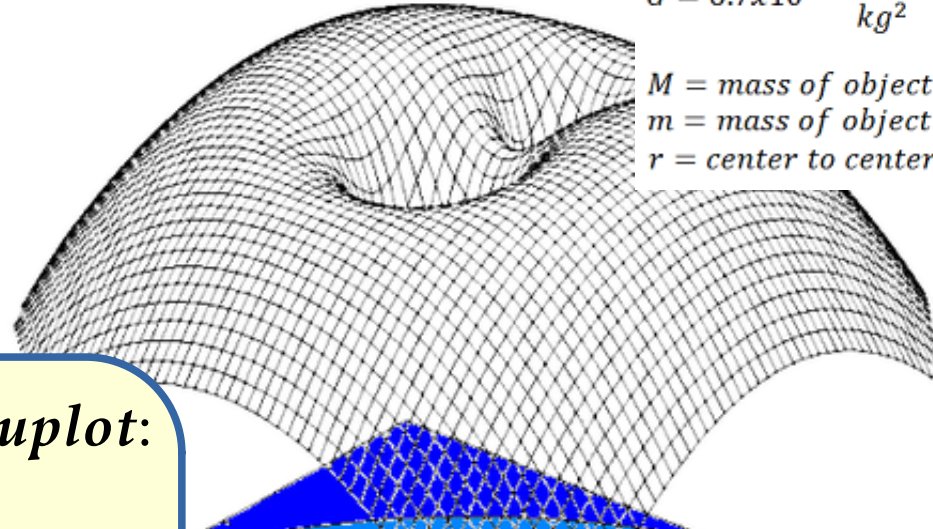
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Try it for yourself! E.g. in *gnuplot*:

$$U(x,y) = (-G*m1/\sqrt{x**2+y**2}) - G*m2/\sqrt{(x-orbsep)**2+y**2})/Rsun$$

`m1=20; m2=15; orbsep=5 #[Msun],[Rsun]`

`G=6.6743*10**(-8); Rsun=6.957*10**(10) #[cgs]`

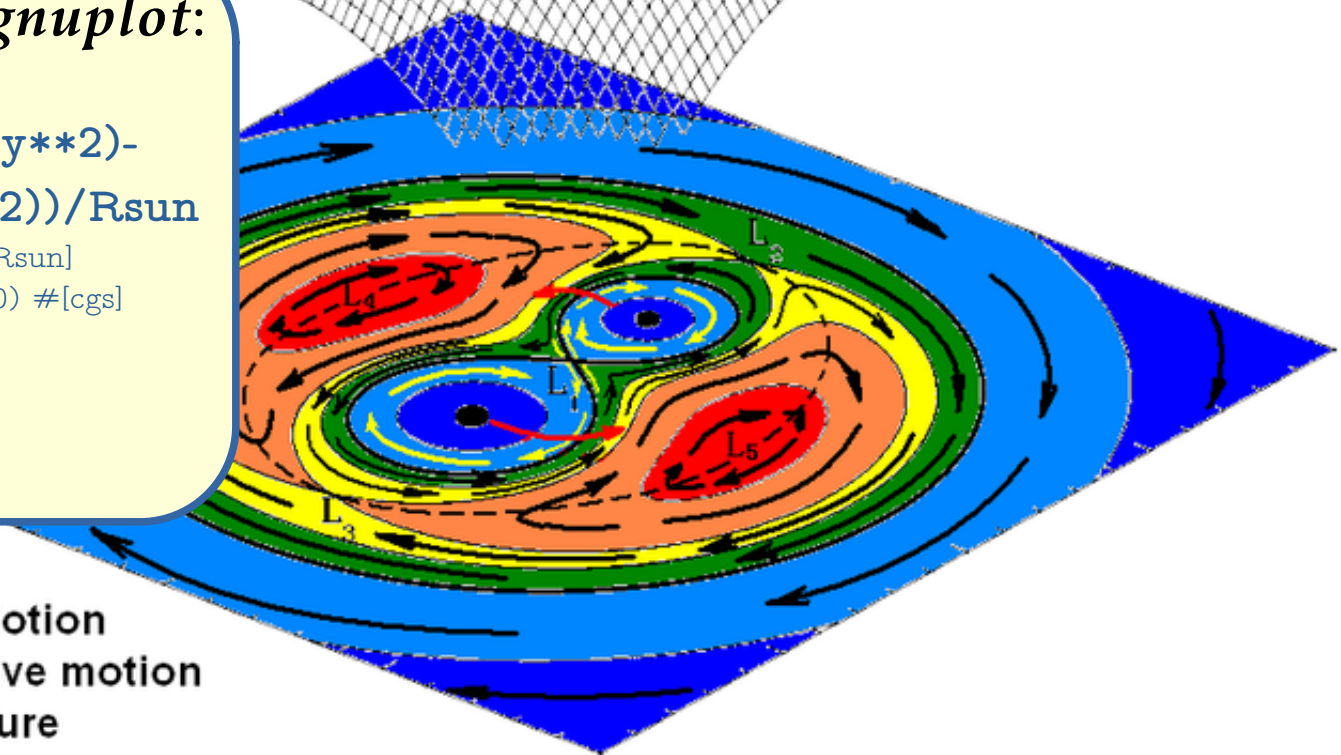
`set isosamples 60,60`

`set pm3d`

`splot U(x,y)`

Legend:

-  revolution
-  relative motion
-  Capture



Roche-lobe facts

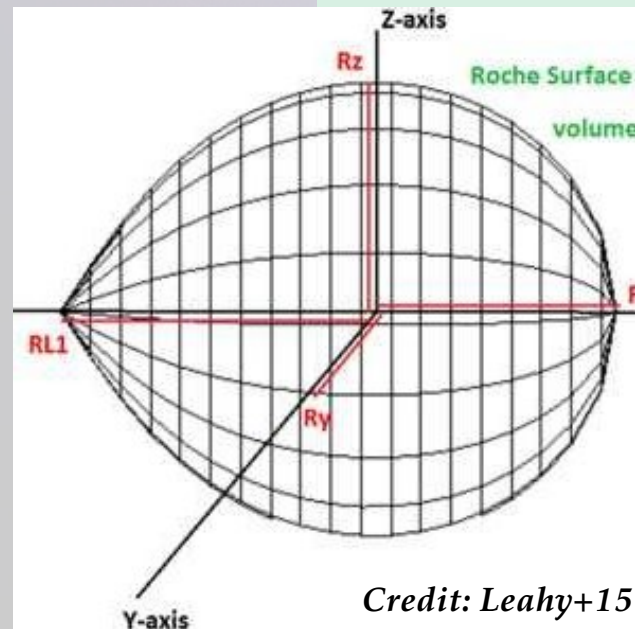
- we can plot it but we cannot explicitly derive it

⇒ approximation of Roche lobe

(Eggleton 1983) $q = m_1/m_2$:
from numerical fit

$$RL_1 = A \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$$

orbital separation: A



Credit: Leahy+15

Roche-lobe facts

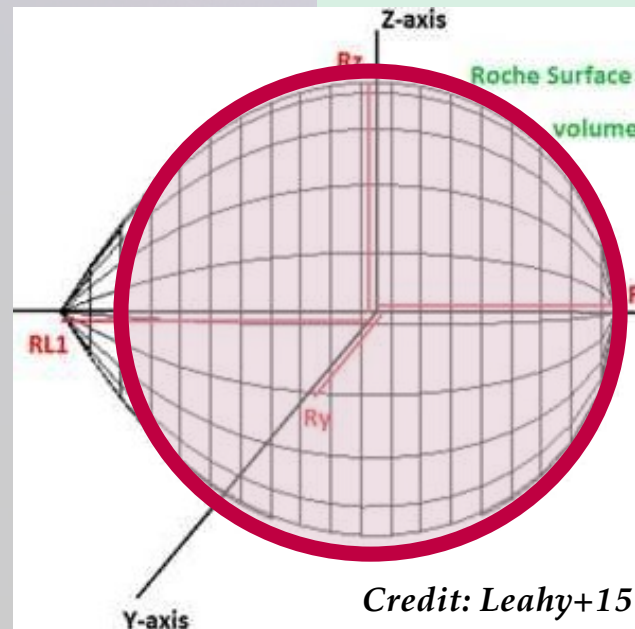
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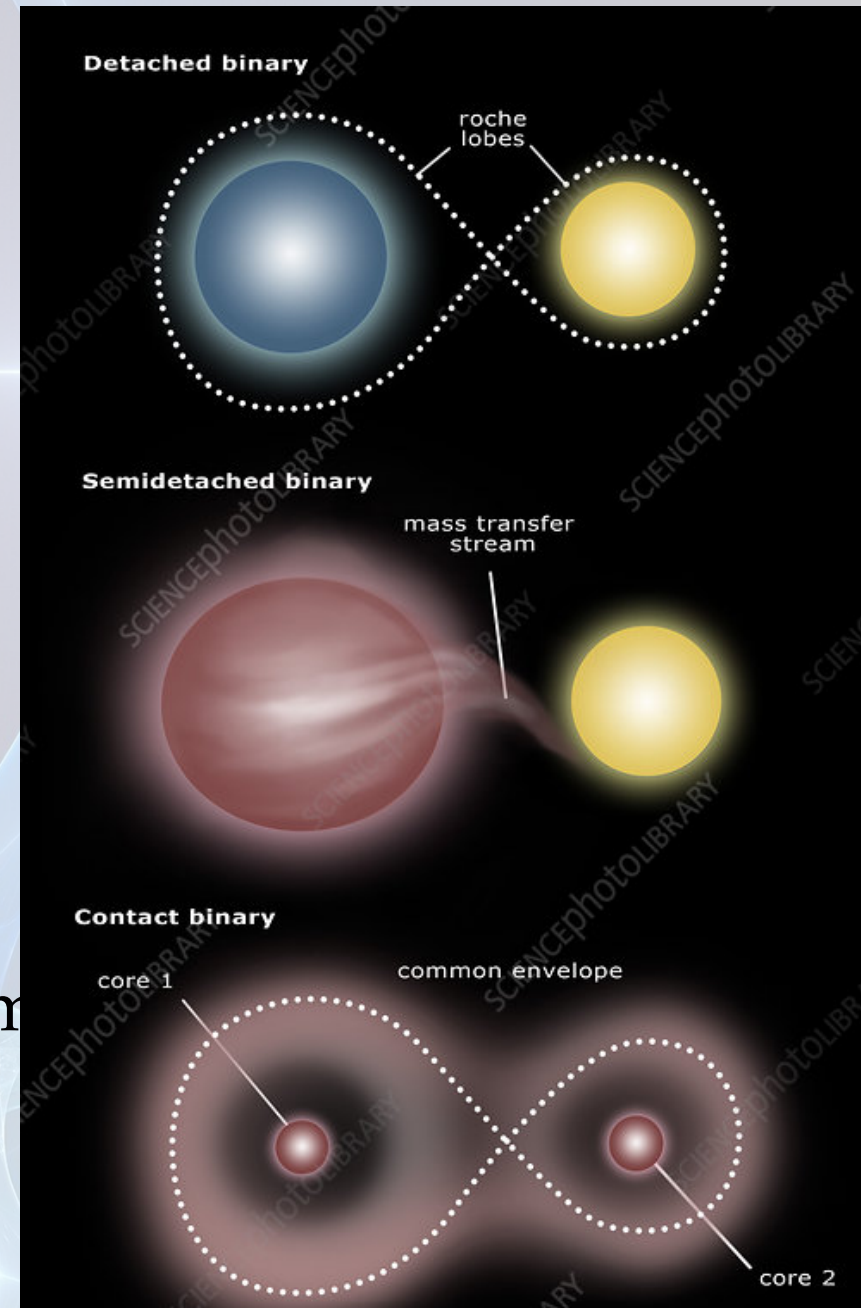
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Why does the Roche-lobe matter?

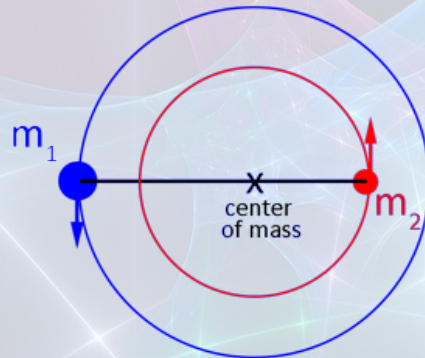
- Mass transfer.
- Some important terms:
 - primary/secondary (companions)
 - donor/accretor **mass gainer**
 - M_1/M_2 losing mass / gaining mass
 - detached system
 - Roche-lobe overflow
 - semi-detached, contact system
 - ‘common envelope’ (...)
 - ▶ *stellar envelope*



Some more terms

- orbital separation = orb. distance
- period = orbital period
 - \neq rotational period!!
(though cf. *synchronization*)
e.g. due to tidal forces
- initial orbital separation *vs.* actual
- initial period *vs.* actual
- Connection between distance & period?

Kepler's 3rd law:



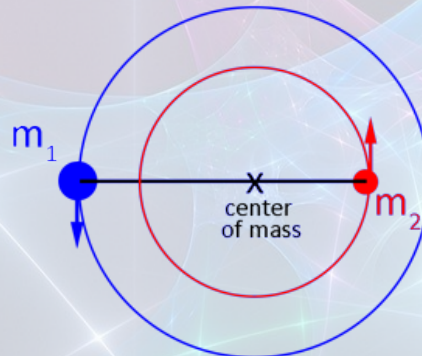
$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} r^3$$

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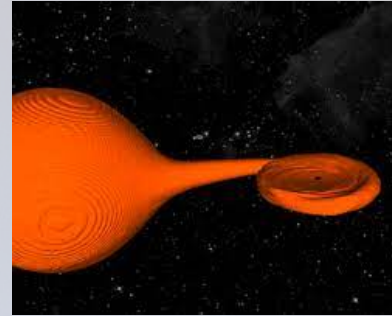
Eccentricity, circular orbit
Inclination (orbital)
Orbital parameters
Semi-major axis 'a'
Semi-minor axis 'b'
Mass ratio, $q = m_2/m_1$
orbital angular momentum

Kepler's 3rd law:



$$P^2 = \frac{4\pi^2}{G(M_1 + M_2)} r^3$$

What happens when the Roche-lobe is overflowed?



- Mass transfer

- ~ mass exchange

- ~ (binary) interaction

Youtube video to watch:

youtube.com/watch?v=xAjq7VGnf4s

Today.

'Case A', 'Case B', 'Case C' mass transfer

- **Historical categorization** (cf. stellar classes O, B, A, F... or supernova classification type Ia, Ib, II...) – **useful to know**
even if its getting outdated

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- case A: MS
- case B: HG
- case C: He-b.
(donor's evolutionary status)

MS = Main Sequence
HG = Hertzsprung-gap
He-b. = helium-burning

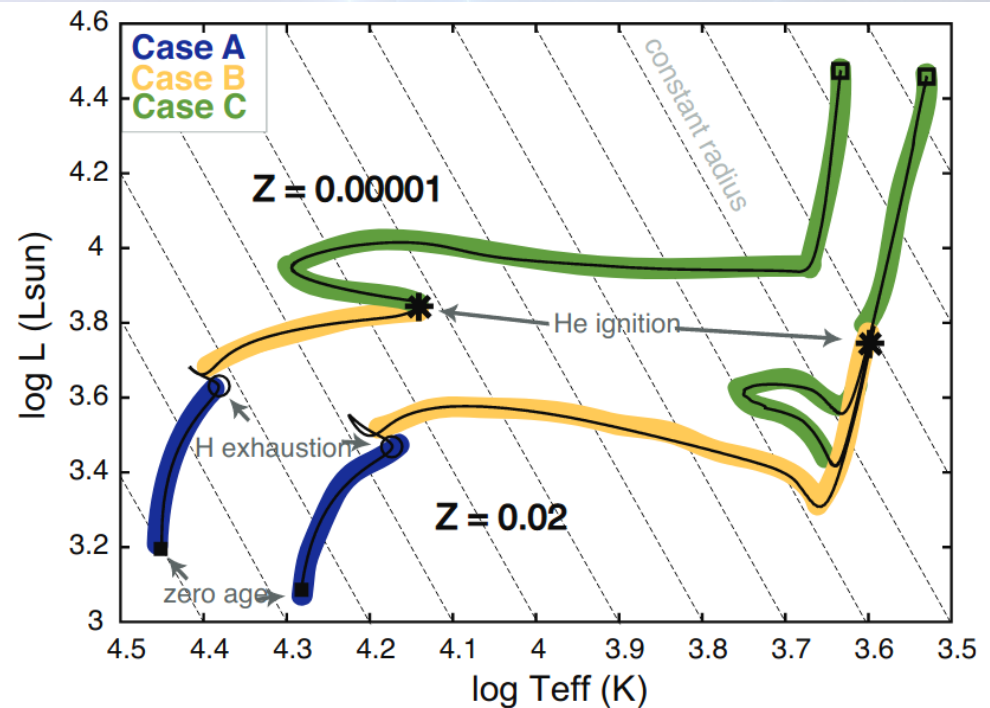


Figure 1.1: Evolutionary tracks in the HR-diagram of a $6 M_{\odot}$ star illustrating the effect of metallicity on the occurrence of the different cases of mass transfer. The dashed diagonal lines indicate lines of constant radii. Cases A, B and C are defined in the text of Section 1.5.1. Figure adapted from De Mink et al. (2008b).

Sub-categories exist...

- case BA: mass transfer is initiated during helium core burning
- case BB: initiated after helium core burning is terminated, but before the ignition of carbon

- case B: HG
- case C: He-b.
(donor's evolutionary status)

MS = Main Sequence
 HG = Hertzsprung-gap
 He-b. = helium-burning

mass transfer

stellar classes O, B, A, F... or

useful to know
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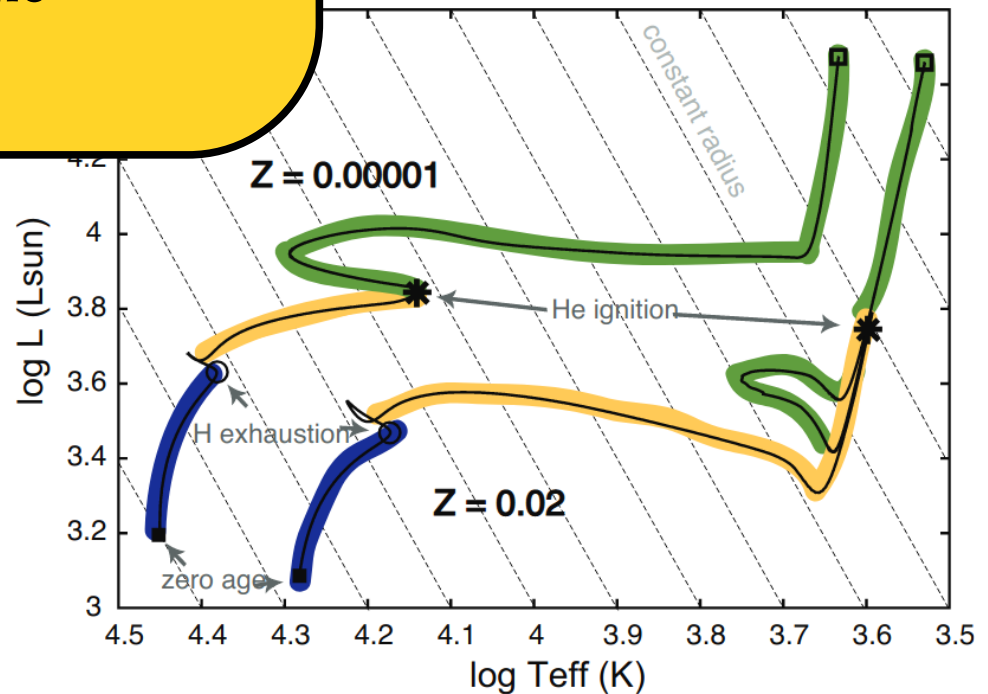


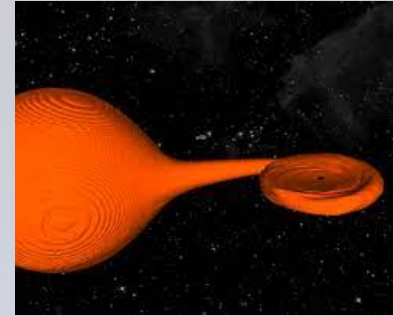
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Sidenote: TIMESCALES

- The **dynamical timescale**. How long would it take for the star to expand or contract if the balance between pressure gradients and gravity was suddenly disrupted? Same as the “free-fall time scale”. For the Sun, it is about half an hour.
- The **thermal timescale**. Also known as the Kelvin-Helmholtz timescale. Suppose nuclear reactions were suddenly cut off in the star (but the stability somehow stays intact). The thermal timescale is the time required for the star to radiate all its reservoir of thermal energy away. For a Sun-like star the thermal timescale is ~ 10 Myr.
- The **nuclear timescale**. This is the evolutionary timescale of a star. As the star evolves the composition of the core changes due to nuclear burning. The nuclear timescale is the time for the star to change its core composition by a factor of order unity. For a Sun-like star the nuclear timescale is ~ 10 Gyr.

$$\tau_{\text{nuc}} \gg \tau_{\text{KH}} \gg \tau_{\text{dyn}}$$

What happens when the Roche-lobe is overflowed?



- Mass transfer

- ~ mass exchange

- ~ (binary) interaction

Youtube video to watch:

youtube.com/watch?v=xAjq7VGnf4s

Material of the star becomes unbound. It might flow off.
(outer layers)

If it does, where does it end up?

(1) – on the top of the companion (“transfer”)

(2) – in the surroundings (non-conservative mass “transfer”)

in reality: a mix of (1)+(2) or some other option (e.g. an accretion/decretion disc?)
((disc: circumstellar or circumbinary))

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atch:
f4s

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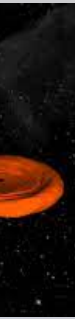
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...



atch:
f4s

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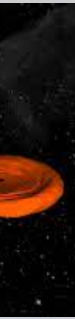
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What happens to the donor after having lost some layers?

...

How does the orbit (and thus the Roche-lobe) change?

...



atch:
f4s

Orbital evolution during mass transfer

- suppose conservative mass transfer:
 - orbit shrinks if $M_{\text{donor}} > M_{\text{acc}}$
 - orbit expands if $M_{\text{donor}} < M_{\text{acc}}$

cf. prof. Onno Pols'
lecture notes on binaries
[\[LINK\]](#)

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- if the mass transfer is non-conservative:
 - then we also need to take into account how much angular momentum is lost from the system...

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- Roche-lobe is effected:

⇒ approximation of Roche lobe
(Eggleton 1983) $q = m_1/m_2$

$$RL_1 = A \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})}$$

orbital separation: A

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- Roche-lobe is effected:

⇒ approximation of Roche lobe
(Eggleton 1983) $q = m_1/m_2$

- And remember:
massive stars have
WINDS...

$$RL_1 = A \frac{0.49 q^{2/3}}{0.6 q^{2/3} + \ln(1 + q^{1/3})}$$

orbital separation: A

and winds carry away ang.mom. too

What happens to the donor after losing layers?

- Can the donor regain its stability after RLOF?
 - if yes: *stable* mass transfer – or detachment
(depending also on RL-evolution)
 - if no: *unstable* mass transfer (🤪)

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- Can the donor regain its stability after RLOF?
 - if yes: *stable* mass transfer – or detachment (depending also on RL-evolution)
 - if no: *unstable* mass transfer (🙄)
- Stable mass transfer:
 - donor remains in thermal equilibrium while continuing mass transfer driven by stellar evolution related expansion (or by orbital shrinkage due to ang. mom. loss)
 - donor does not remain in thermal eq. but the mass transfer may still be stable, driven (self-regulatingly) by thermal readjustment of the donor

hardcore
stuff

$$\tau_{\text{nuc}} \gg \tau_{\text{KH}} \gg \tau_{\text{dyn}}$$

What happens to the donor after losing layers?

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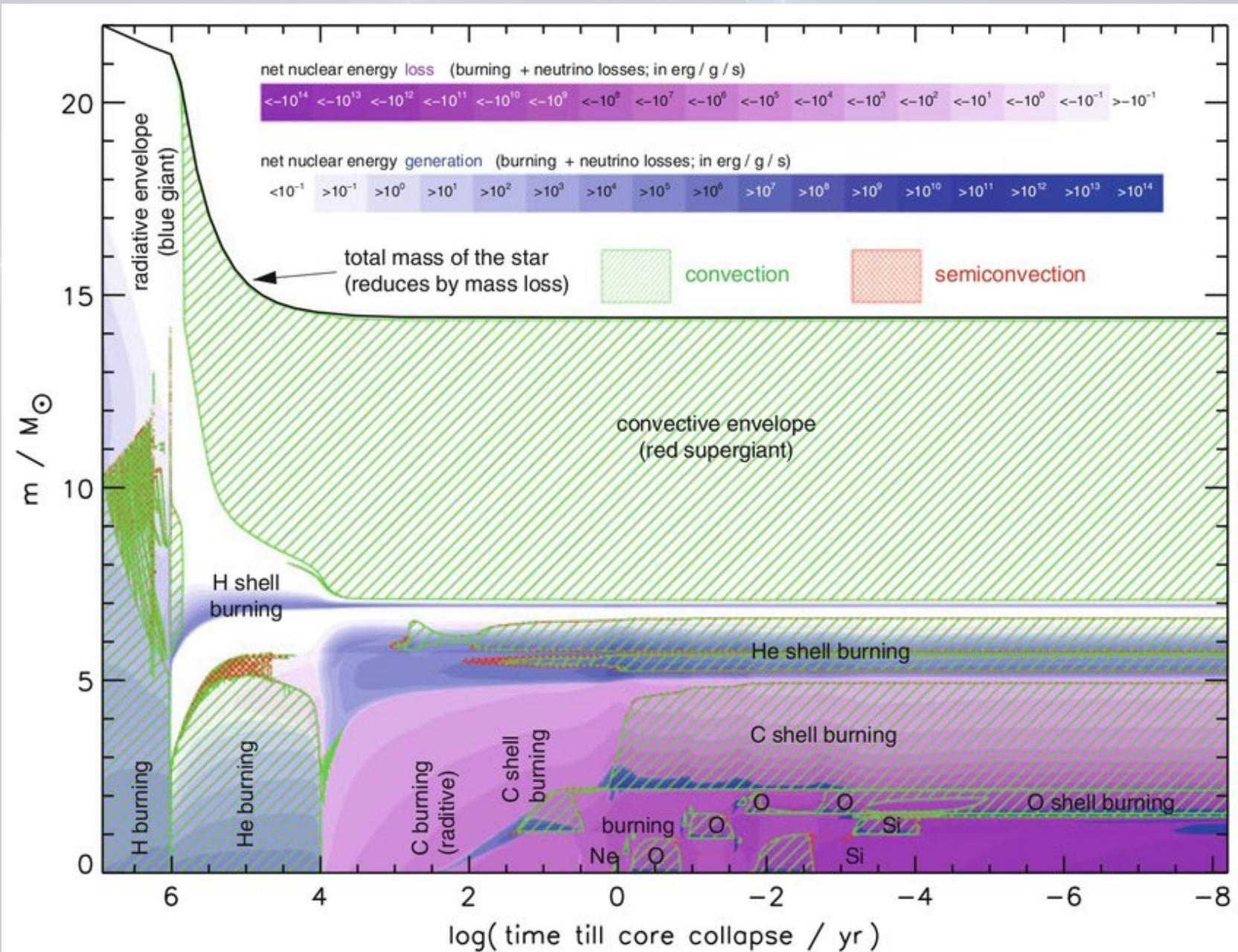
- donor remains in thermal equilibrium while continuing mass transfer related expansion (or by ...)

Detailed calculations show that stars with **radiative envelopes** shrink rapidly (τ_{dyn}) in response to mass loss, while stars with **convective envelopes** tend to expand or keep a roughly constant radius (τ_{KH}).

hardcore stuff

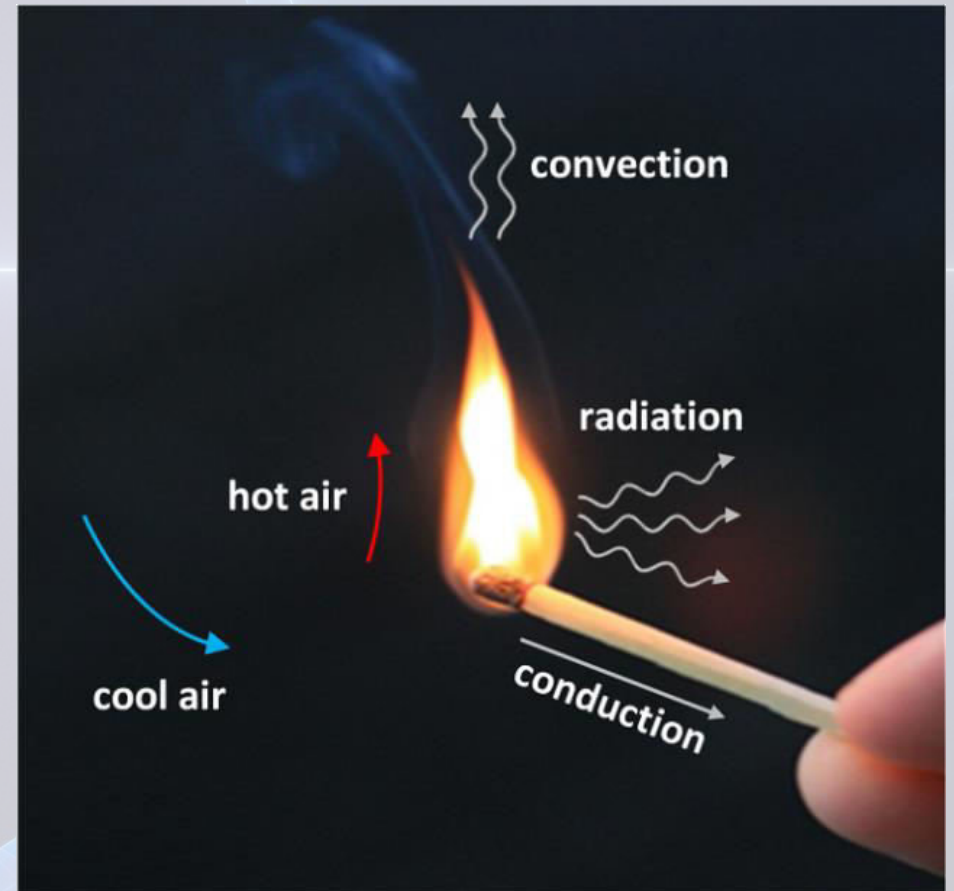
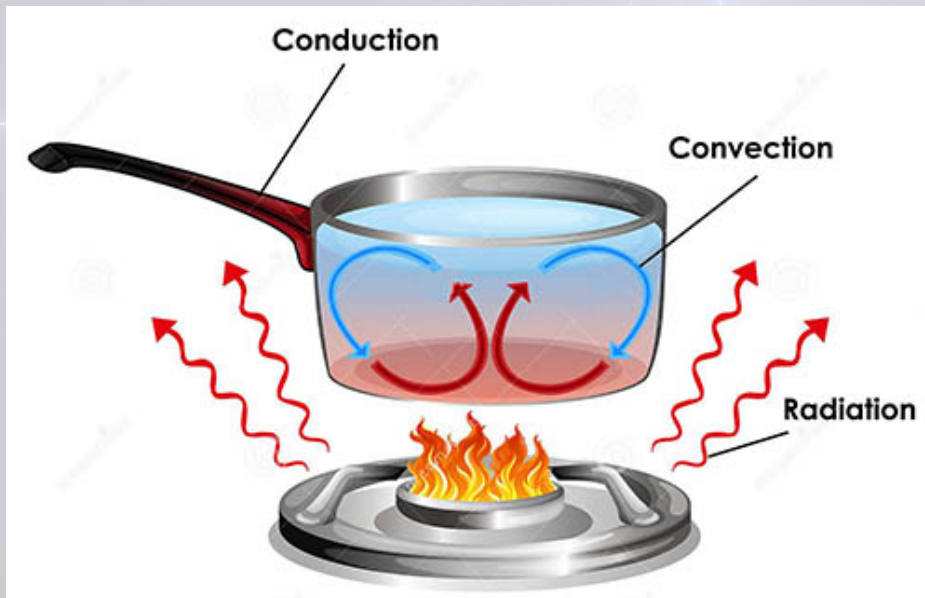
$$\tau_{\text{nuc}} \gg \tau_{\text{KH}} \gg \tau_{\text{dyn}}$$

Remeinder: Kippenhahn diagram



Remeinder: convection

and about *heat transfer* in general



- convection arises wherever heat needs to be transported extra efficiently
e.g. burning core of massive stars, envelope of (super)giants and low-mass stars...
- leads to strong mixing (cf. boiling soup)

Reminder: convection

and about *heat transfer* in general

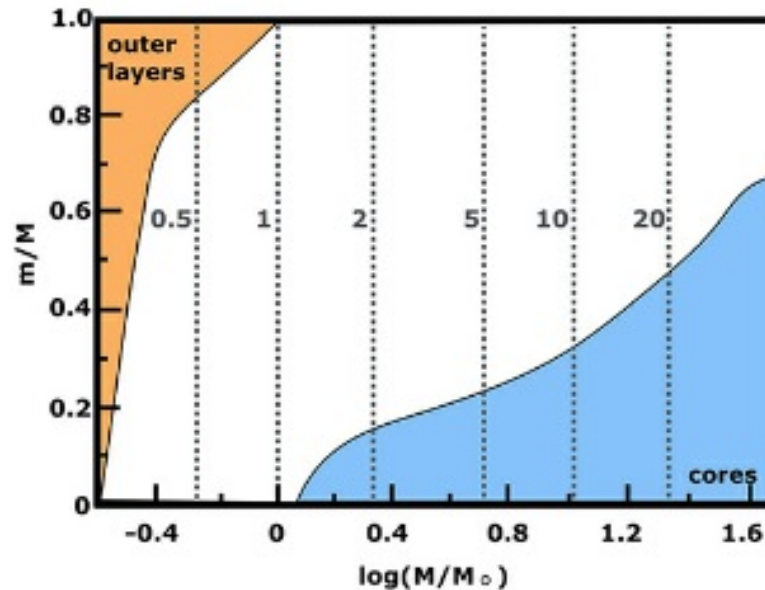
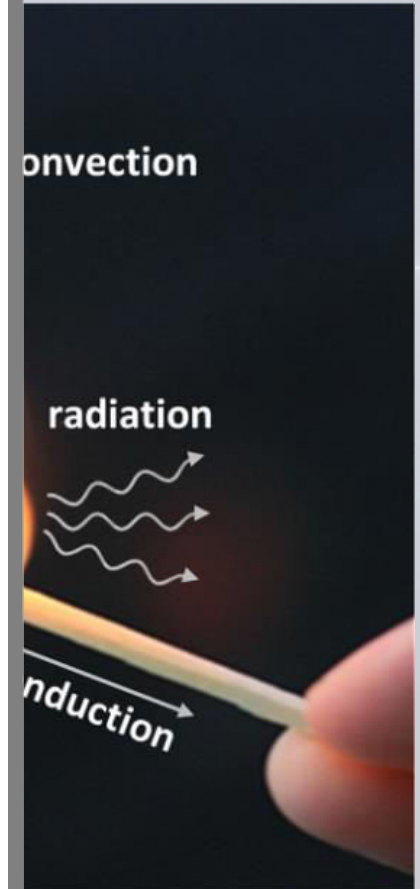


Figure 7.6. Occurrence of convection in stars at the beginning of the core H-fusion phase (ZAMS). The mass of convective envelopes (orange) and convective cores (blue) is expressed as a fraction of the stellar mass, from $m/M = 0$ in the core to $m/M = 1$ at the surface. The vertical lines indicate the stel-

- convection arises when there is a temperature gradient, e.g. burning core
- leads to strong mixing



...tly
...and low-mass stars...

Remeinder: convection

and about *heat transfer* in general



Make sure to remember:

- massive stars's cores are convective
(the Sun's core is radiative!)
- supergiants' (aka post-MS massive stars')
envelope is *also* convective
(will be important later, in binary interactions)

convection

radiation

conduction

... and low-mass stars...

What happens to the donor after losing layers?

- Can the donor regain its stability after RLOF?
 - if yes: *stable* mass transfer – or detachment (depending also on RL-evolution)
 - if no: *unstable* mass transfer

- Stable mass transfer:

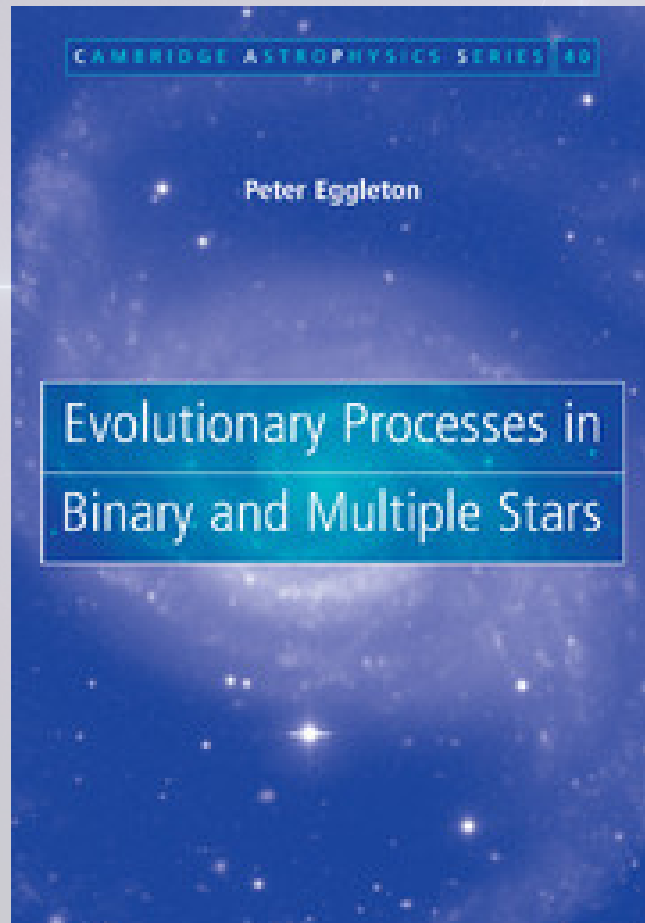
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Detailed calculations show that stars with **radiative envelopes** shrink rapidly (τ_{dyn}) in response to mass loss, while stars with **convective envelopes** tend to expand or keep a roughly constant radius (τ_{KH}).

hardcore
stuff

$$\tau_{\text{nuc}} \gg \tau_{\text{KH}} \gg \tau_{\text{dyn}}$$

Further reading:



- Peter Eggleton:
Evolutionary Processes in Binary and Multiple Stars
(2006, Cambridge University Press)

cf. prof. Onno Pols'
lecture notes on binaries
[[LINK](#)]

Unstable mass transfer

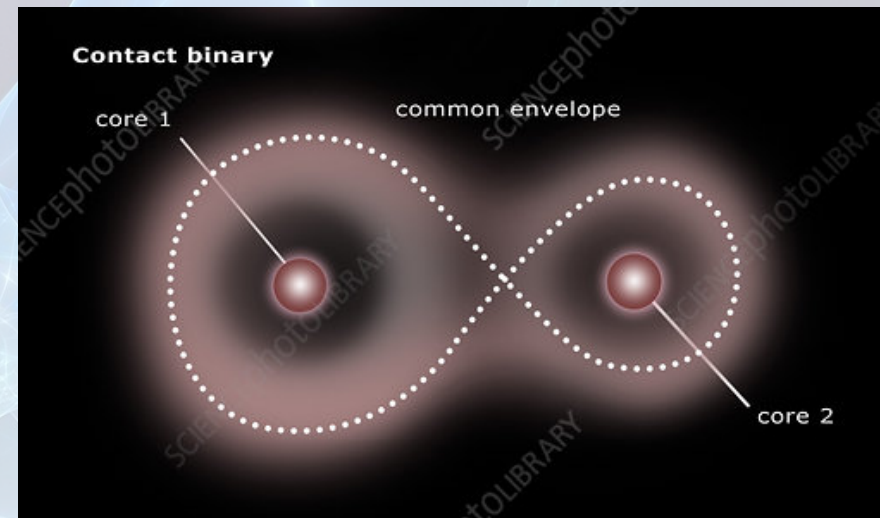


Unstable mass-transfer

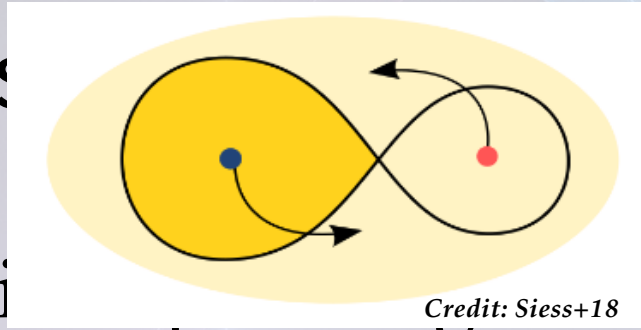


- if the donor is expanding too quickly (τ_{dyn}) and thus cannot stay within its Roche lobe: ever-increasing mass-transfer rates
- this is an unstable, runaway situation secondary cannot accrete fast enough
- has dramatic effects: “common envelope” situation

$$\tau_{\text{nuc}} \gg \tau_{\text{KH}} \gg \tau_{\text{dyn}}$$

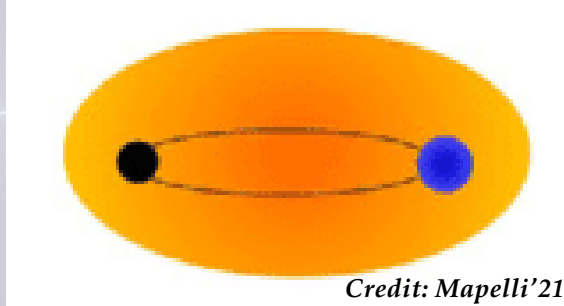


Unstable mass-transfer



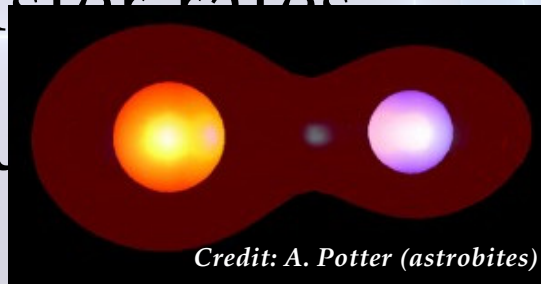
Credit: Siess+18

- if the donor is $\tau_{KH} \ll \tau_{nuc}$ quickly (τ_{KH}) and thus cannot stay within its Roche lobe:

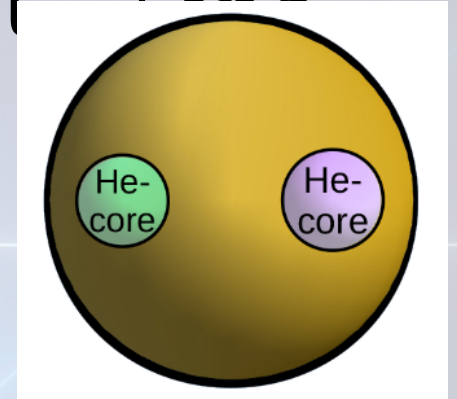


Credit: Mapelli'21

- mass-transfer rates



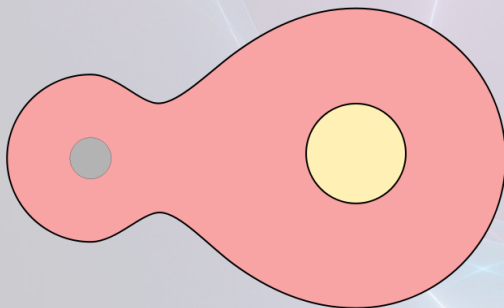
Credit: A. Potter (astrobit.es)



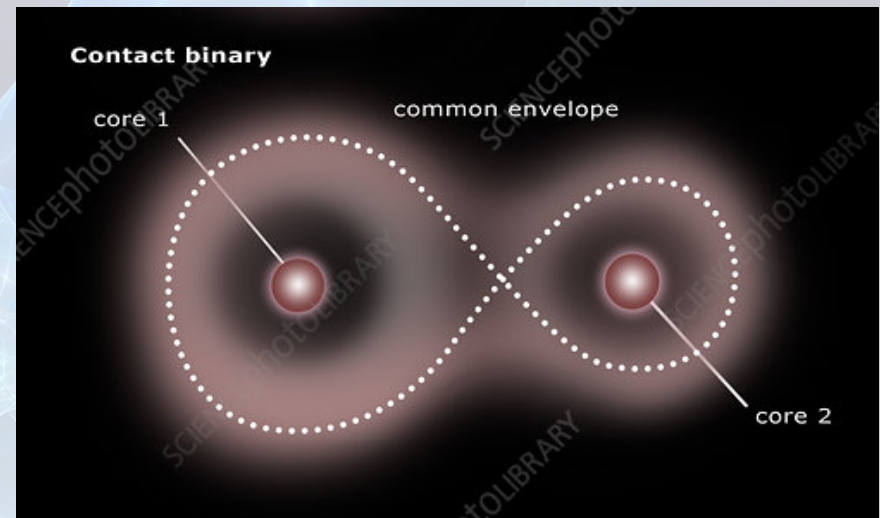
Credit: Yours Truly ;) [Vigna-Gomez+18]

- stable, run on

- has dramatic effects: “common envelope” situation



Credit: Wikipedia



$$\tau_{nuc} \gg \tau_{KH} \gg \tau_{dyn}$$

What we know about CE

- short lived phase
 - observed?? how??

Movies :)

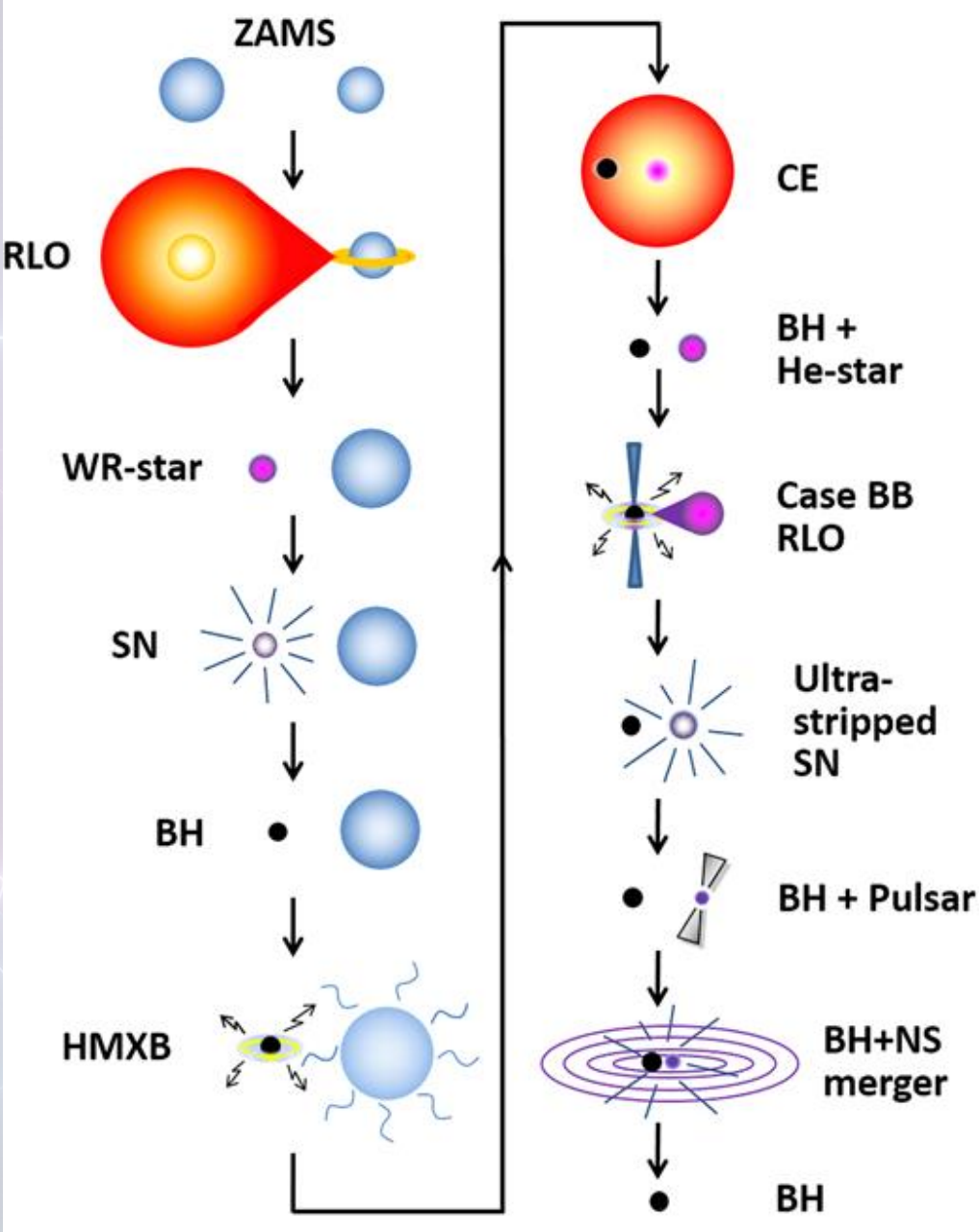
Passy+12:
0.88 M_⊙ (RG)
+ 0.15 M_⊙
companion

Moreno+21:
10 M_⊙ (RSG) +
BH
companion

- but it probably occurs
 - explaining close white dwarf-binaries
(WD=ex-Red Giant: no other way to get that close)
- 3D simulations are still very expensive
 - in practice: derived relations between
orbital energy & binding energy of the envelope
- Result: envelope is (probably?) ejected due to friction. (If not: merger. *No GW possible.*)
of the two stellar cores

Let's play!

Let's play!



Let's play!

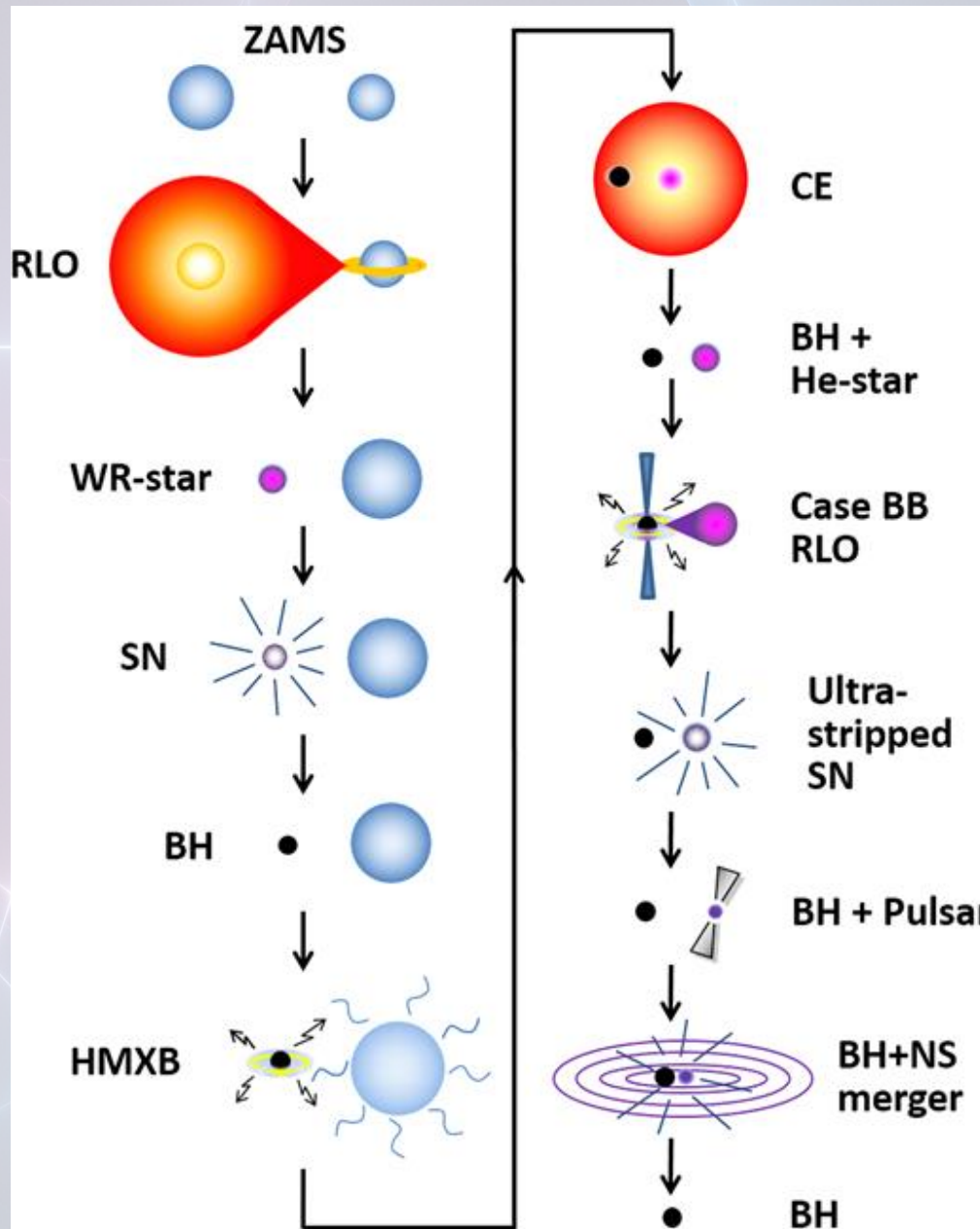
Zero-age Main Seq.

Roche-lobe overflow:
stable mass transfer

Wolf-Rayet star
(naked He-star with
strong emission lines)

Supernova may kick out
the companion! Survival
rate?

Accreting black hole:
High-Mass X-ray Binary
(observed: periodic
pulsations in X-rays)



Common Envelope!



Probably a HMXB?

Stripped = type Ib
Ultra-stripped = type Ic

(Pulsar: a rotating,
magnetized neutron star)

GRAV. WAVES!!!

Credit: Kruckow+18

This is just one possible scenario, actually.

There are more.

It's play!

stable mass transfer

RLO



WR-star



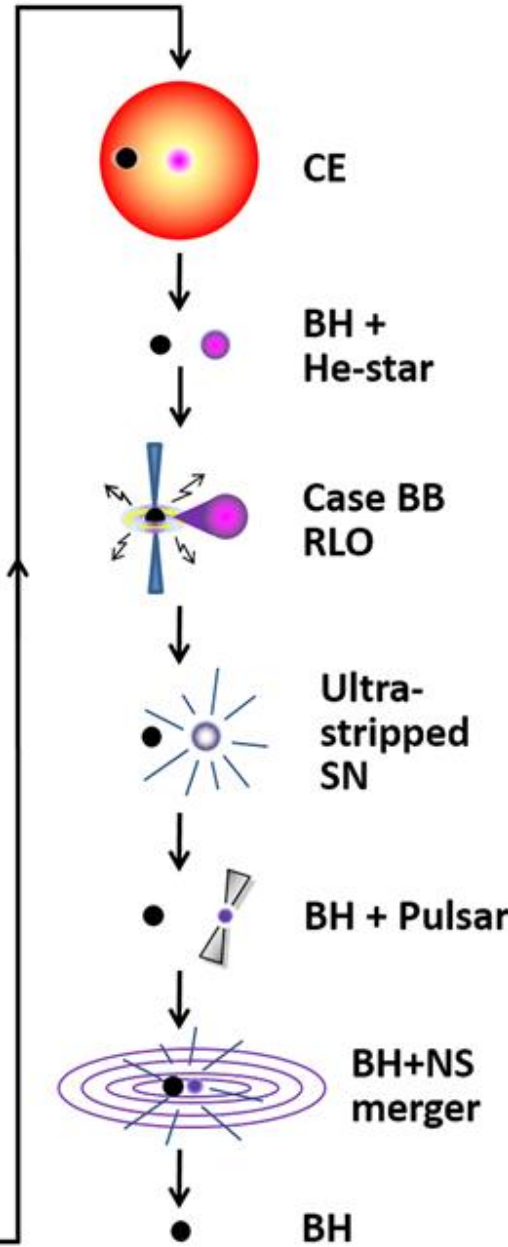
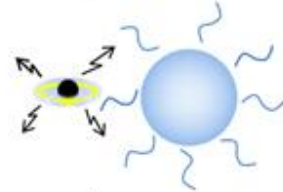
SN



BH



HMXB



CE

BH + He-star

Case BB RLO

Ultra-stripped SN

BH + Pulsar

BH+NS merger

BH

Common Envelope!



Probably a HMXB?

Stripped = type Ib
Ultra-stripped = type Ic

(Pulsar: a rotating, magnetized neutron star)

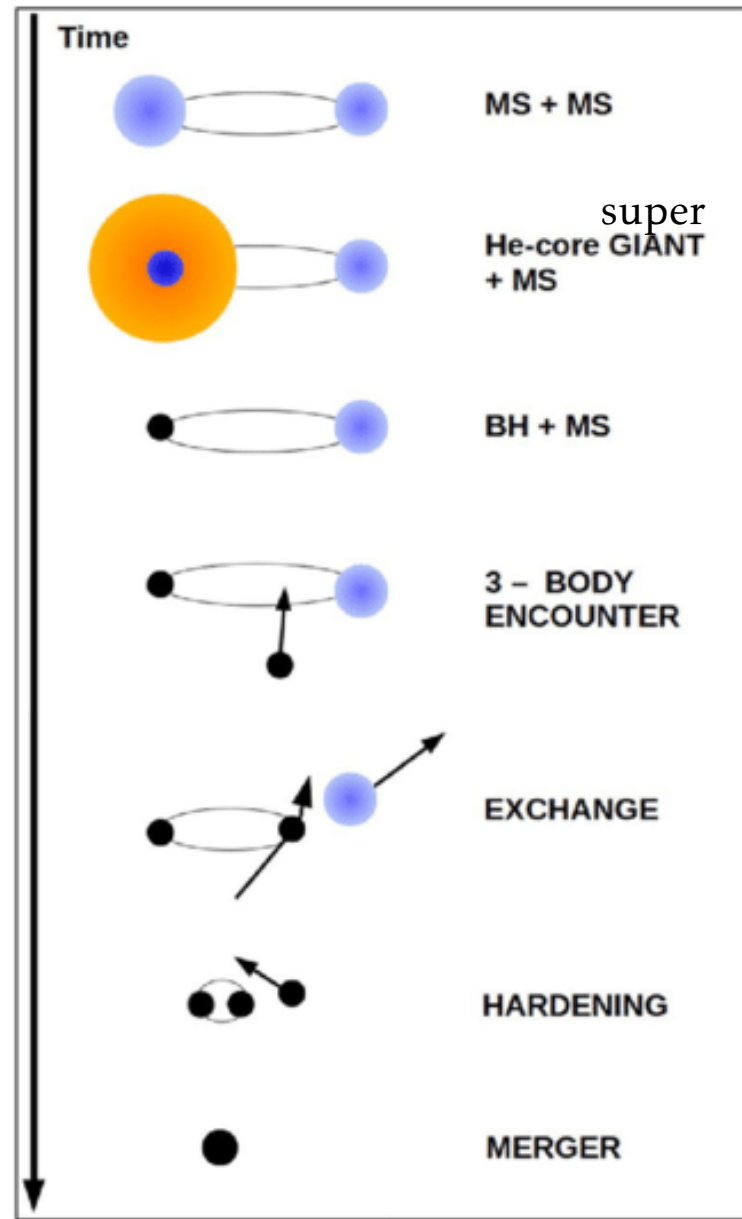
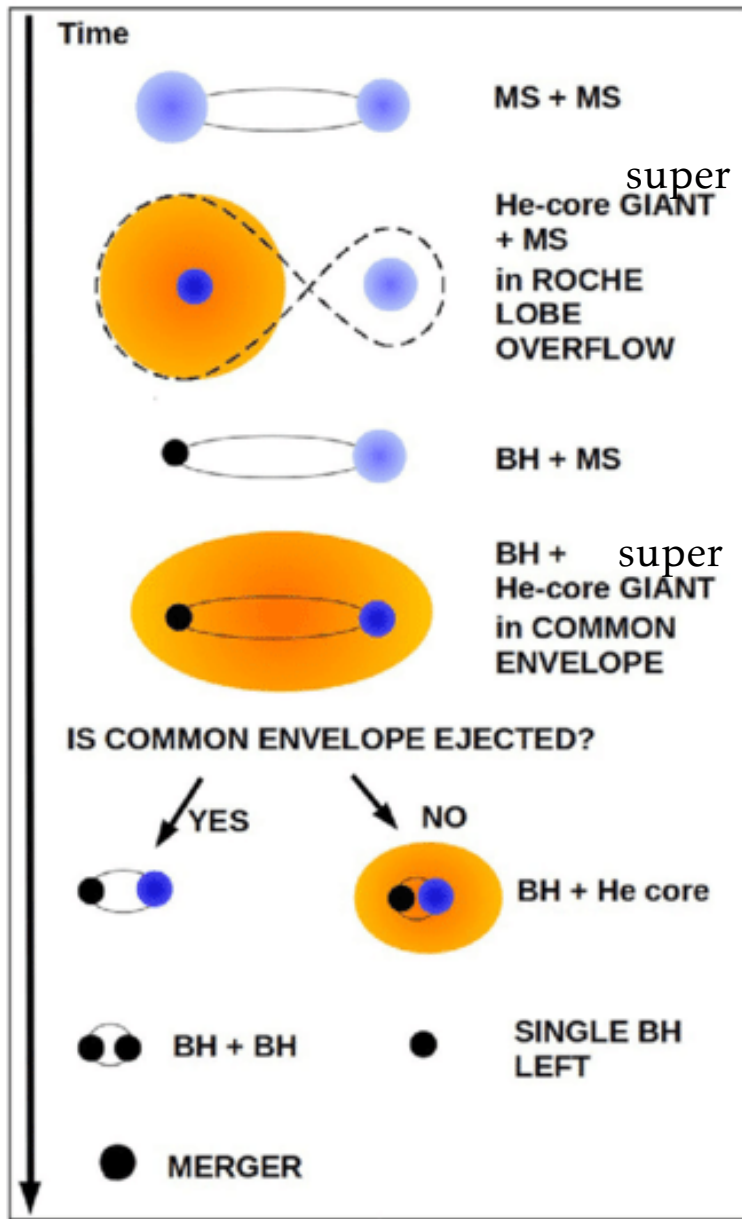
GRAV. WAVES!!!

Credit: Kruckow+18

Supernova may kick out the companion! Survival rate?

Accreting black hole: High-Mass X-ray Binary (observed: periodic pulsations in X-rays)

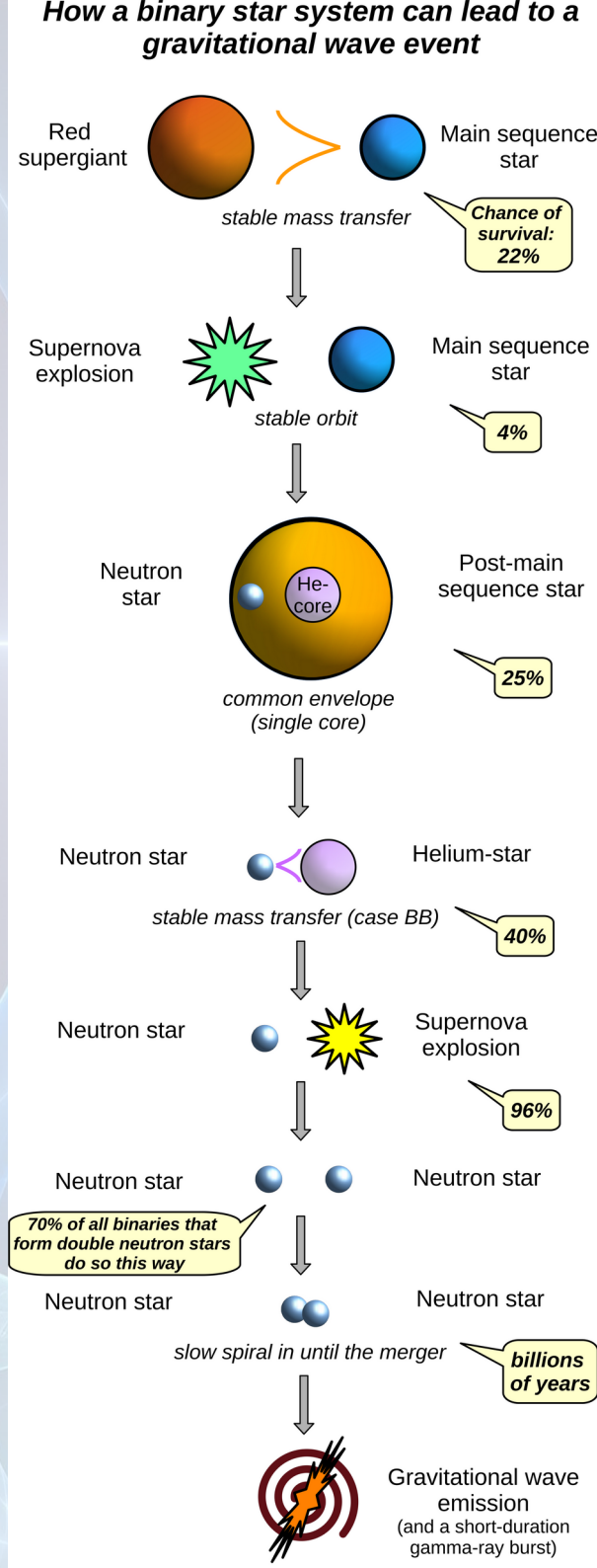
Some other scenarios...



a triple!

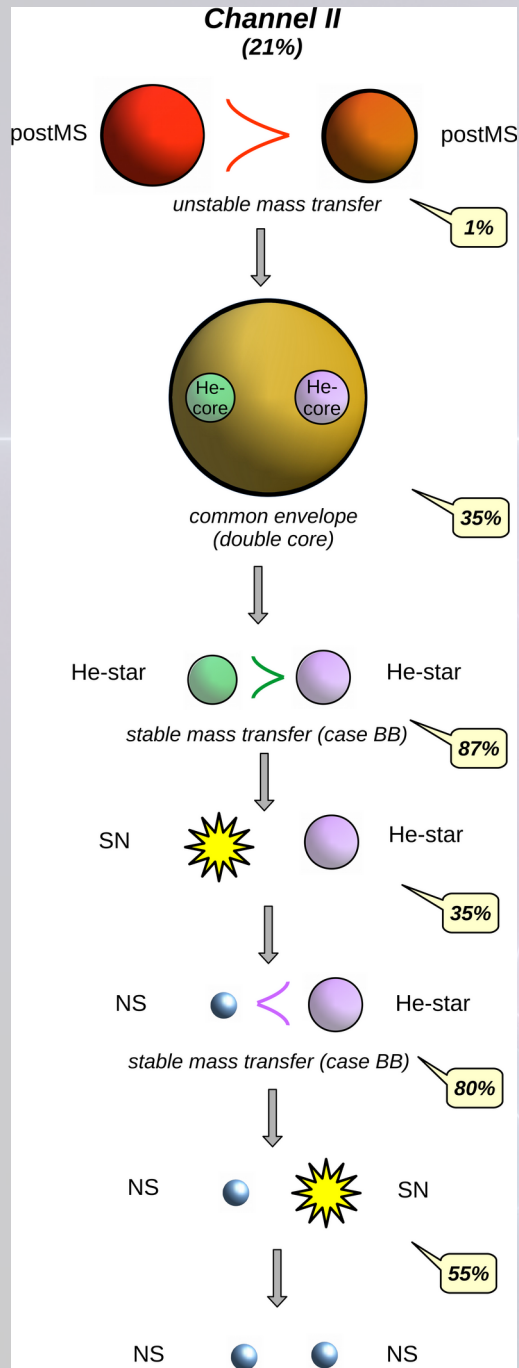
= orbit shrinks

There are more... :D

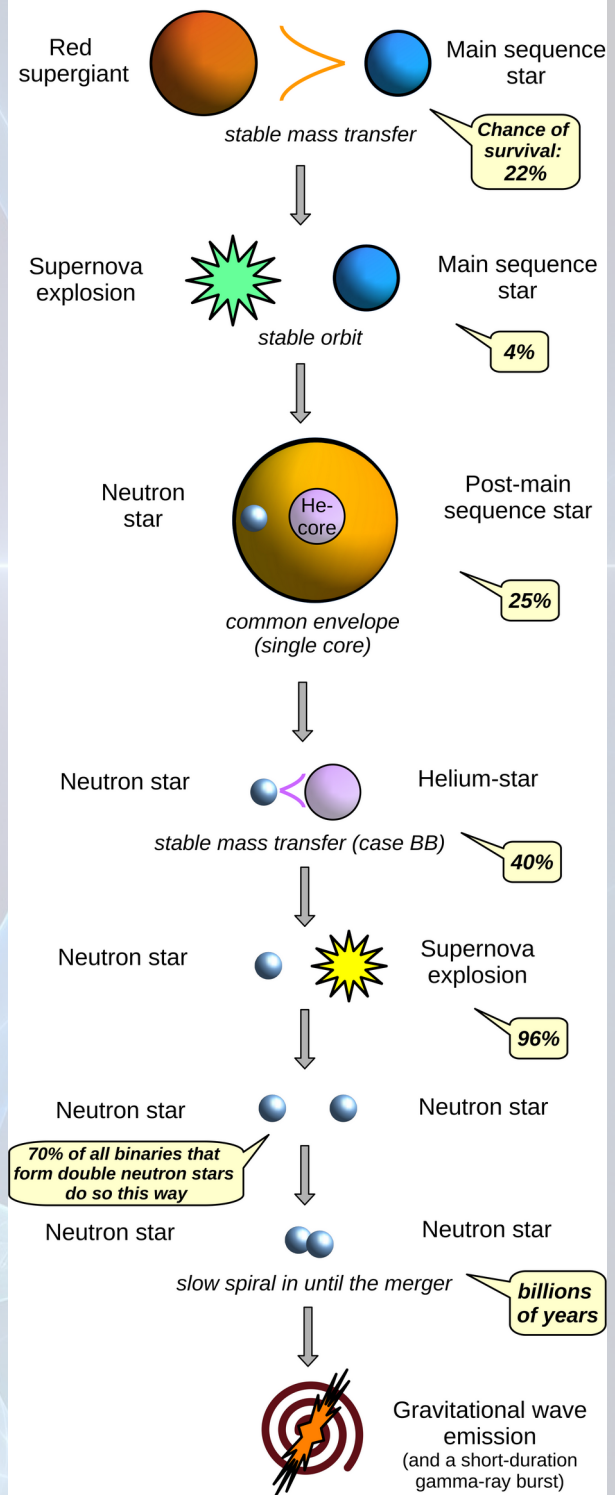


Credit: Vigna-Gomez+18

There are more... :D

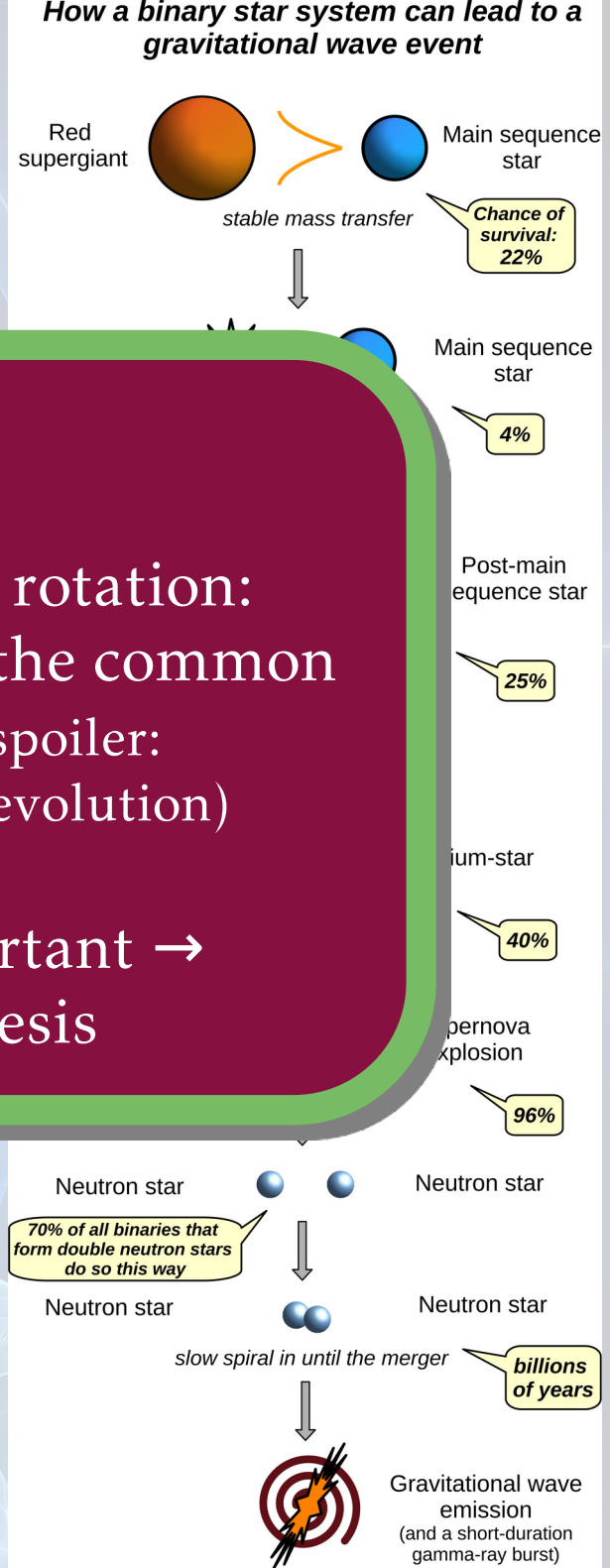
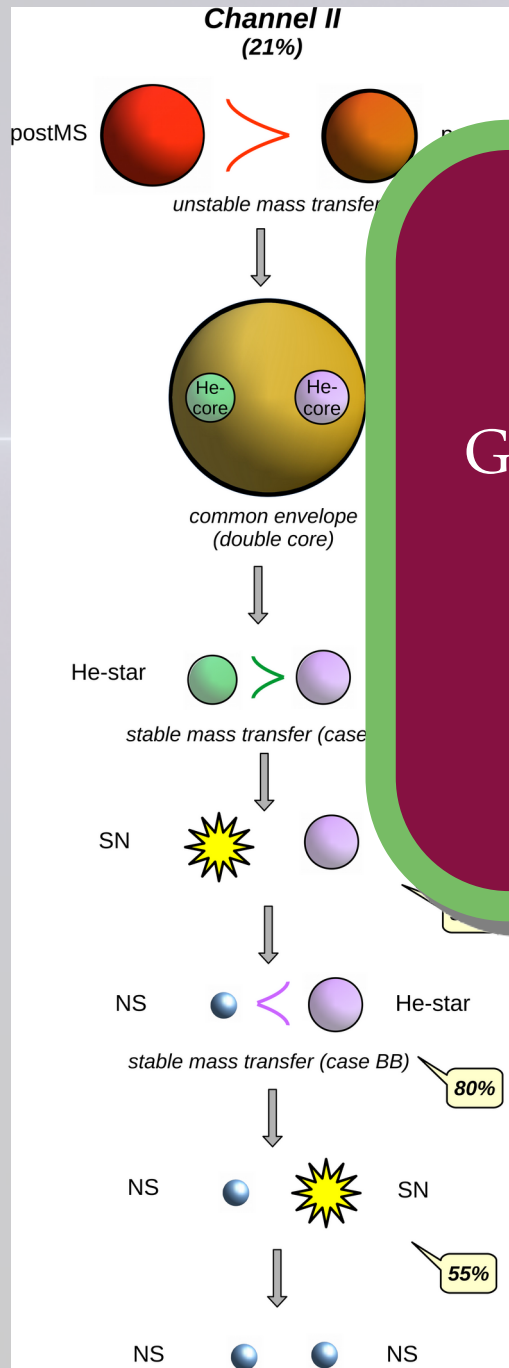


How a binary star system can lead to a gravitational wave event



Credit: Vigna-Gomez+18

There are more... :D



Next time:

effects of metallicity & rotation:
 GW-progenitors without the common envelope scenario (spoiler: chemically homogeneous evolution)

why statistics is important → population synthesis