Phenomenology of inhomogenous cosmological spacetimes in scalefree variables

The Gowdy case

Lars Andersson

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Reference: L. Andersson, H. van Elst and C. Uggla, Gowdy phenomenology in scale-invariant variables, *to appear in* A Spacetime Safari: Papers in Honour of Vincent Moncrief, special issue of Classical and Quantum Gravity.

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Gowdy spacetimes

Spatially compact, orthogonally transitive G_2 spacetimes, with group $\mathrm{U}(1) \times \mathrm{U}(1)$ are also known as Gowdy spacetimes. The possible spatial topologies are S^3, $S^2 \times S^1$, $\mathbb{T}^3$. We will consider only the vacuum case.

The Gowdy spacetimes are the simplest inhomogenous spacetimes. We wish to understand the global properties of Gowdy spacetimes, as part of the effort to understand the global structure of cosmological models. The issues of interest include

- Cosmic censorship
- BKL/AVTD behavior at singularity
- asymptotics in the expanding direction

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- (Berger and Moncrief, 1993), (Berger and Garfinkle, 1998): Numerical studies, showed spiky features, indicated AVTD behavior
- (Kichenassamy and Rendall, 1998): constructed AVTD solutions using Fuchsian method
- (Rendall and Weaver, 2001): constructed solutions with spikes using Gowdy–Ernst transformations
- (Ringström, 2002; Ringström, 2003), (Chae and Chruściel, 2003): proved "small data" results for Gowdy singularity: if energy density bounded, AVTD holds. C& C: AVTD holds on open and dense subset of S^1

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Dynamical systems approach

(Uggla et al., 2003): use scale invariant frame variables:

- first order form of evolution equations
- classify fixed point sets, attractors etc.
- natural formulation of BKL proposal
- asymptotic dynamical system

Here, consider Hubble normalized frame variables

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Metric form of Gowdy equations

Gowdy metric on $\mathbb{T}^3 \times \mathbb{R}$:

$$\ell_0^{-2}\,\mathrm{d}s^2 = e^{(t-\lambda)/2}\,(-\,e^{-2t}\,\mathrm{d}t^2 + \mathrm{d}x^2) \\ +\,e^{-t}\,[\,e^{P}\,(\mathrm{d}y_1 + Q\,\mathrm{d}y_2)^2 + e^{-P}\,\mathrm{d}y_2^2\,]\ ,$$

Evolution equations:

$$P_{,tt} - e^{-2t}P_{,xx} = e^{2P}\left(Q_{,t}^2 - e^{-2t}Q_{,x}^2\right)\ ,$$
$$Q_{,tt} - e^{-2t}Q_{,xx} = -\,2\left(P_{,t}Q_{,t} - e^{-2t}P_{,x}Q_{,x}\right)\ ,$$

Constraints:

$$0 = \lambda_{,t} - \left[P_{,t}^2 + e^{-2t}P_{,x}^2 + e^{2P}(Q_{,t}^2 + e^{-2t}Q_{,x}^2)\right]\ ,$$
$$0 = \lambda_{,x} - 2\left(P_{,t}P_{,x} + e^{2P}Q_{,t}Q_{,x}\right)\ .$$

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$\mathbb{H}^2$ picture of Gowdy

(P,Q) are coordinates on $\mathbb{H}^2$, the Teichmüller space of the flat metrics on the $\mathbb{T}^2$, with metric $g_{\mathbb{H}^2} = \mathrm{d}P^2 + e^{2P}\,\mathrm{d}Q^2$. Therefore Gowdy spacetime corresponds to a curve of loops in $\mathbb{H}^2$.

The coordinate representation of $\mathbb{H}^2$ corresponds to a choice of infinity in the ball model. Those points of the loop which travel to the infinity point induce a sharp feature in the (P,Q) representation.

For general solutions, cusps develop on the loop, the cusp points travel faster in the $\mathbb{H}^2$ than the rest of the loop, these are “spike points”.

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- "False spikes" (gauge effects); sharp local minima in P, with $P < 0$
- "True spikes"; sharp local maxima in P, correspond to zeros in Q_x.

The Gowdy field equations is a wave–map like system for $u : S^1 \times \mathbb{R} \to \mathbb{H}^2$, (with $\tau = e^{-t}$)

$$-\partial_\tau^2 u - \frac{1}{\tau}\,\partial_\tau u + \partial_x^2 u + \Gamma(\partial u \cdot \partial u) = 0 \ ,$$

Hyperbolic velocity v:

$$v := ||\partial_t u||^2_{\mathbb{H}^2} = \sqrt{P_{,t}^2 + e^{2P}\,Q_{,t}^2}\ .$$

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energy density

$$\lambda_{,t}(t,x) = ||\partial_t u||^2_{\mathbb{H}^2} + e^{-2t}\,||\partial_x u||^2_{g_{\mathbb{H}^2}}\ ,$$

where the second term is expected to tend to zero pointwise.

Therefore expect that $\lambda_{,t}$ tends to a limit, the square asymptotic hyperbolic velocity $\hat{v}^2$, where

$$\hat{v}(x) := \lim_{t\to+\infty} v(t,x)\ ,$$

By the analysis of Rendall and Weaver, for low velocity spikes $\hat{v}(x_{\text{spike}}) = 1 + \kappa$, while for nearby points, $\hat{v}(x) = 1 - \kappa$ for $\kappa \in (0,1)$.

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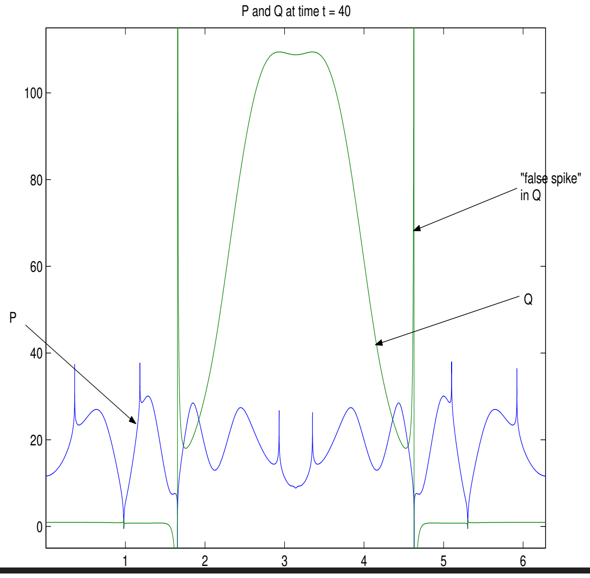
P and Q at $t = 40$



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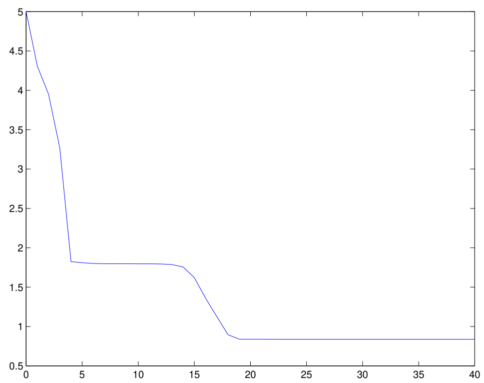
Time evolution of $\max_x v$



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frame

Introduce a group-invariant orthonormal frame $\{\, \boldsymbol{e}_a \,\}_{a=0,1,2,3}$, with $\boldsymbol{e}_0$ timelike, $\boldsymbol{e}_1$ defining the direction of spatial inhomogeneity, and with $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ tangent to the G_2-orbits; align $\boldsymbol{e}_2$ with one of the Killing fields.

$$\boldsymbol{e}_0 = N^{-1}\,\partial_t\,, \quad \boldsymbol{e}_1 = {e_1}^1\,\partial_x\,, \quad \boldsymbol{e}_2 = {e_2}^2\,\partial_{y_1}\,, \quad \boldsymbol{e}_3 = {e_3}^2\,\partial_{y_1} + {e_3}^3\,\partial_{y_2}\,,$$

Explicitely,

$$\begin{aligned}
\boldsymbol{e}_0 &= N^{-1}\,\partial_t &&= \ell_0^{-1} e^{(3t+\lambda)/4}\,\partial_t\,,\\
\boldsymbol{e}_1 &= {e_1}^1\,\partial_x &&= \ell_0^{-1} e^{-(t-\lambda)/4}\,\partial_x\,,\\
\boldsymbol{e}_2 &= {e_2}^2\,\partial_{y_1} &&= \ell_0^{-1} e^{(t-P)/2}\,\partial_{y_1}\,,\\
\boldsymbol{e}_3 &= {e_3}^2\,\partial_{y_1} + {e_3}^3\,\partial_{y_2} &&= \ell_0^{-1} e^{(t+P)/2}\,(-Q\partial_{y_1} + \partial_{y_2})\,.
\end{aligned}$$

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connection variables

The nonzero connection variables are:

- Θ, the volume expansion rate, $H := \frac{1}{3}\,\Theta$,
- σ_{11}, σ_{22}, σ_{33} and σ_{23} (with $0 = \sigma_{11} + \sigma_{22} + \sigma_{33}$); shear,
- n_{22} and n_{23}, commutation functions, giving the spatial connection on $\mathbb{T}^3$,
- $\dot{u}_1$, the acceleration of the integral curves of $\boldsymbol{e}_0$,
- Ω_1, the angular velocity of rotation of $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ about $\boldsymbol{e}_1$.

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Introduce variables σ_+, σ_-, $\sigma_\times$, n_- and $n_\times$:

$$\sigma_+ = \frac{1}{2}\,(\sigma_{22} + \sigma_{33}) = -\,\frac{1}{2}\,\sigma_{11}\ , \qquad \sigma_- = \frac{1}{2\sqrt{3}}\,(\sigma_{22} - \sigma_{33})\ ,$$

$$\sigma_\times = \frac{1}{\sqrt{3}}\,\sigma_{23}\ , \qquad n_- = \frac{1}{2\sqrt{3}}\,n_{22}\ ,$$

$$n_\times = \frac{1}{\sqrt{3}}\,n_{23}\ .$$

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<u>**Hubble normalized variables**</u>

$$(\,\mathcal{N}^{-1},\,E_1{}^1\,) := (\,N^{-1},\,e_1{}^1\,)/H$$

$$(\,\Sigma_{\dots},\,N_{\dots},\,\dot{U},\,R\,) := (\,\sigma_{\dots},\,n_{\dots},\,\dot{u}_1,\,\Omega_1\,)/H\ .$$

State vector:

$$\boldsymbol{X} = (E_1{}^1, \Sigma_+, \Sigma_-, N_\times, \Sigma_\times, N_-)^T\ .$$

To write evolution equations, need deceleration parameter q and logarithmic spatial Hubble gradient r,

$$(q+1) := -\,\mathcal{N}^{-1}\,\partial_t \ln(\ell_0 H)\ ,$$

$$r := -\,E_1{}^1\,\partial_x \ln(\ell_0 H)\ ,$$

where q and r satisfy the integrability condition

$$\mathcal{N}^{-1}\,\partial_t r - E_1{}^1\,\partial_x q = (q + 2\Sigma_+)\,r - (r - \dot{U})\,(q+1)\ .$$

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- The area density $\mathcal{A}$ has timelike gradient if $(1+\Sigma_+)^2 > 0$. Use gauge
$$\mathcal{N} = \frac{C}{(1+\Sigma_+)} \ , \quad C \in \mathbb{R}\backslash\{0\} \ ,$$
Then
$$\mathcal{A} = \ell_0^2 e^{2Ct} \ ,$$
and t is the dimensionless *area time coordinate*.
- $C = -\frac{1}{2}$ corresponds to the metric form of Gowdy.
- In terms of the Hubble normalized variables, the hyperbolic velocity v becomes
$$v = \sqrt{3}\,\frac{\sqrt{\Sigma_-^2 + \Sigma_\times^2}}{(1+\Sigma_+)} \ .$$

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Hubble normalized equations

Constraints:

$$\begin{aligned} \text{Gauge:} \quad & (r - \dot{U}) = E_1{}^1\,\partial_x \ln(1+\Sigma_+) \ , \\ \text{Gauss:} \quad & 1 = \Sigma_+^2 + \Sigma_-^2 + \Sigma_\times^2 + N_\times^2 + N_-^2 \ , \\ \text{Codazzi:} \quad & \dot{U} = -\,3\,\frac{(N_\times\,\Sigma_- - N_-\,\Sigma_\times)}{(1+\Sigma_+)} \ . \end{aligned}$$

Let
$$q = 2(\Sigma_+^2 + \Sigma_-^2 + \Sigma_\times^2) - \frac{1}{3}\,(E_1{}^1\,\partial_x - r + \dot{U})\,\dot{U} \ .$$

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Evolution equations

$$
\begin{aligned}
C^{-1}(1+\Sigma_+)\,\partial_t E_1{}^1 &= (q+2\Sigma_+)\,E_1{}^1 \ , \\
C^{-1}(1+\Sigma_+)\,\partial_t (1+\Sigma_+) &= (q-2)\,(1+\Sigma_+) \ , \\
C^{-1}(1+\Sigma_+)\,\partial_t \Sigma_- + E_1{}^1\,\partial_x N_\times &= (q-2)\,\Sigma_- + (r-\dot{U})\,N_\times \\
&\quad + 2\sqrt{3}\,\Sigma_\times^2 - 2\sqrt{3}\,N_-^2 \ , \\
C^{-1}(1+\Sigma_+)\,\partial_t N_\times + E_1{}^1\,\partial_x \Sigma_- &= (q+2\Sigma_+)\,N_\times + (r-\dot{U})\,\Sigma_- \ , \\
C^{-1}(1+\Sigma_+)\,\partial_t \Sigma_\times - E_1{}^1\,\partial_x N_- &= (q-2-2\sqrt{3}\Sigma_-)\,\Sigma_\times \\
&\quad - (r-\dot{U}+2\sqrt{3}N_\times)\,N_- \ , \\
-E_1{}^1\,\partial_x \Sigma_\times &= (q+2\Sigma_+ + 2\sqrt{3}\Sigma_-)\,N_- \\
&\quad - (r-\dot{U}-2\sqrt{3}N_\times)\,\Sigma_\times \ .
\end{aligned}
$$

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Conversion formulae

$$
H = -\,\ell_0^{-1} e^{(3t+\lambda)/4}\,\frac{3+\lambda_{,t}}{12} \ .
$$

$$
\begin{aligned}
\left(\mathcal{N}^{-1},\, E_1{}^1\right) &= -\,12\left(\frac{1}{3+\lambda_{,t}},\, e^{-t}\,\frac{1}{3+\lambda_{,t}}\right) \ , \\
\left(\Sigma_+,\, \dot{U}\right) &= \left(\frac{3-\lambda_{,t}}{3+\lambda_{,t}},\, 3\,e^{-t}\,\frac{\lambda_{,x}}{3+\lambda_{,t}}\right) \ , \\
(\Sigma_-,\, N_\times) &= 2\sqrt{3}\left(-\frac{P_{,t}}{3+\lambda_{,t}},\, e^{-t}\,\frac{P_{,x}}{3+\lambda_{,t}}\right) \ , \\
(\Sigma_\times,\, N_-) &= -\,2\sqrt{3}\left(e^{P}\,\frac{Q_{,t}}{3+\lambda_{,t}},\, e^{-(t-P)}\,\frac{Q_{,x}}{3+\lambda_{,t}}\right) \ ,
\end{aligned}
$$

$$
q = 2 + \frac{12\lambda_{,tt}}{(3+\lambda_{,t})^2} \ , \qquad r = e^{-t}\,\frac{3}{3+\lambda_{,t}}\left(\lambda_{,x} + \frac{4\lambda_{,tx}}{3+\lambda_{,t}}\right) \ .
$$

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Invariant sets

- Polarized Gowdy:
$$0 = \Sigma_\times = N_- \ ,$$
- Spatially homogeneous (SH) vacuum spacetimes of class A,
$$\mathbf{0} = \partial_x \boldsymbol{Y} \ , \qquad 0 = \dot{U} = r \ ,$$
with $\boldsymbol{Y} := (\Sigma_+, \Sigma_-, N_\times, \Sigma_\times, N_-)^T$
- Silent boundary (SB):
$$0 = {E_1}^1 \ .$$

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The silent boundary

- For $\boldsymbol{X}$ on SB, we have *unrestricted* spatial dependence. Therefore the SB set is not invariant under the full Einstein evolution equations.
- One expects the Gowdy dynamics to be *asympotic* to the Silent boundary dynamics. We have
$$|{E_1}^1| \leq 4e^{-t} \ .$$
Therefore expect the terms ${E_1}^1 \partial_x \boldsymbol{X}$ to become irrelevant as $t \to \infty$, so that the SB evolution equations give the ***asymptotic dynamical system*** for Gowdy spacetimes.
- The dynamical equations for SB are the same as the ones for the SH system, but the constraints differ. The SB system has a "polarized" subsystem $0 = \Sigma_\times = N_-$, corresponding to the polarized AVTD system studied by Isenberg and Moncrief.

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SB Evolution equations

$$C^{-1}(1+\Sigma_+)\,\partial_t(1+\Sigma_+) = (q-2)\,(1+\Sigma_+)\ ,$$
$$C^{-1}(1+\Sigma_+)\,\partial_t\Sigma_- = (q-2)\,\Sigma_- + 2\sqrt{3}\,\Sigma_\times^2 - 2\sqrt{3}\,N_-^2\ ,$$
$$C^{-1}(1+\Sigma_+)\,\partial_t N_\times = (q+2\Sigma_+)\,N_\times\ ,$$
$$C^{-1}(1+\Sigma_+)\,\partial_t\Sigma_\times = (q-2-2\sqrt{3}\Sigma_-)\,\Sigma_\times - 2\sqrt{3}\,N_\times\,N_-\ ,$$
$$C^{-1}(1+\Sigma_+)\,\partial_t N_- = (q+2\Sigma_+ + 2\sqrt{3}\Sigma_-)\,N_- + 2\sqrt{3}\,N_\times\,\Sigma_\times\ ,$$

with

$$q = 2(\Sigma_+^2 + \Sigma_-^2 + \Sigma_\times^2)\ .$$

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SB Constraints

$$0 = (r - \dot{U})\ ,$$
$$1 = \Sigma_+^2 + \Sigma_-^2 + \Sigma_\times^2 + N_\times^2 + N_-^2\ ,$$
$$\dot{U} = -3\,\frac{(N_\times\,\Sigma_- - N_-\,\Sigma_\times)}{(1+\Sigma_+)}\ .$$

This has as a consequence

$$C^{-1}(1+\Sigma_+)\,\partial_t r = (q+2\Sigma_+)\,r\ .$$

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Comments on SB system

- The variables $r, \dot{U}$ are not dynamical in the SB case.
- The SB equations determine a family of **self–similar** spacetimes.
- Scenario: General spacetime $\rightarrow$ SB, asymptotically locally self–similar $\rightarrow$ attractor in SB system (which is SH??)

  For general G2 or G0, an attractor analogous to the Bianchi II attractor for Bianchi IX is expected to explain the behavior of timelines. This attractor can be studied in the context of the SB system for G2 or G0.

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SH Invariant set

Contains Bianchi class A *except* Bianchi VIII, IX, as these G_3 isometry groups do not contain Abelian G_2 subgroups. The representation differs from the canonical frame representation of Ellis and MacCallum.

SH Evolution equations: the ***same*** as the SB evolution equations.

SH constraints (correspond to $r = \dot{U} = 0$ in SB constraints):

$$1 = \Sigma_+^2 + \Sigma_-^2 + \Sigma_\times^2 + N_\times^2 + N_-^2 \ ,$$
$$0 = N_\times \Sigma_- - N_- \Sigma_\times \ ,$$

Invariant subsets of SH system:

- Taub: $0 = \Sigma_\times = N_\times$, $1 = \Sigma_+^2 + \Sigma_-^2 + N_-^2$
- Kasner: $0 = N_\times = N_-$, $1 = \Sigma_+^2 + \Sigma_-^2 + \Sigma_\times^2$

These combine to explain the main features of the SH and SB dynamics, and of the Gowdy timelines for late times.

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Compare: Bianchi evolution equations

$$N_1' = (q - 4\Sigma_+)N_1,$$
$$N_2' = (q + 2\Sigma_+ + 2\sqrt{3}\Sigma_-)N_2,$$
$$N_3' = (q + 2\Sigma_+ - 2\sqrt{3}\Sigma_-)N_3,$$
$$\Sigma_+' = -(2 - q)\Sigma_+ - 3S_+,$$
$$\Sigma_-' = -(2 - q)\Sigma_- - 3S_-,$$

where $q = 2(\Sigma_+^2 + \Sigma_-^2)$,

$$S_+ = \frac{2}{3}[(N_2 - N_3)^2 - N_1(2N_1 - N_2 - N_3)],$$
$$S_- = \frac{2}{\sqrt{3}}(N_3 - N_2)(N_1 - N_2 - N_3).$$

The Hamiltonian constraint:

$$1 = \Sigma_+^2 + \Sigma_-^2 + [N_1^2 + N_2^2 + N_3^2 - 2(N_1N_2 + N_2N_3 + N_3N_1)].$$

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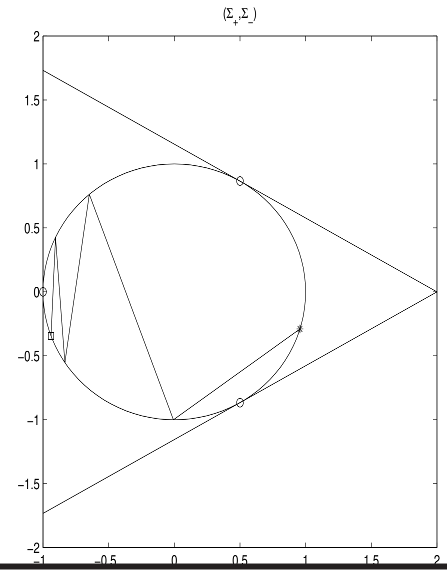
Bianchi VII$_0$ orbit ($N_3 = 0$)



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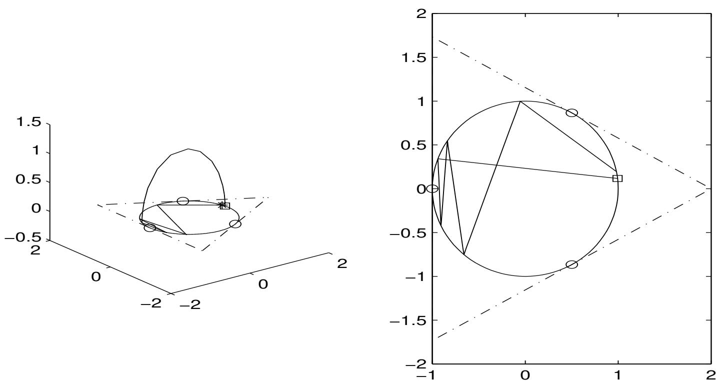
Bianchi IX orbit



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Bianchi II attractor

The Kasner billiard behavior for Bianchi IX is explained by the Bianchi II attractor variety, (Ringström, 2001),

$$N_1 N_2 = N_2 N_3 = N_3 N_1 = 0$$

This is analogous to the candidate attractor variety identified for Bianchi $\mathrm{VI}_{-1/9}$ (Hewitt et al., 2003) and the candidate attractor for the SB system for G_0 cosmologies (Uggla et al., 2003).

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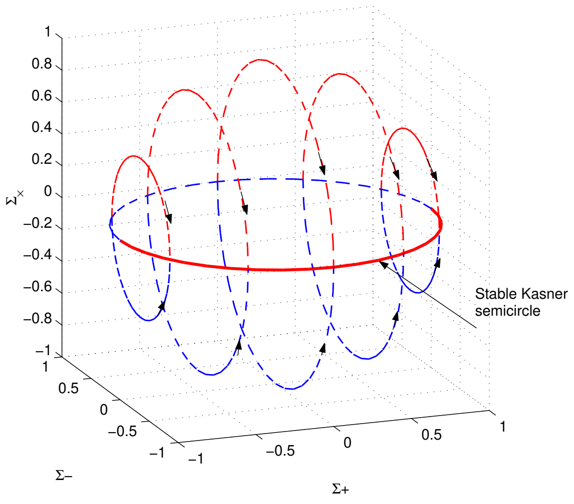
Flow on the Kasner sphere



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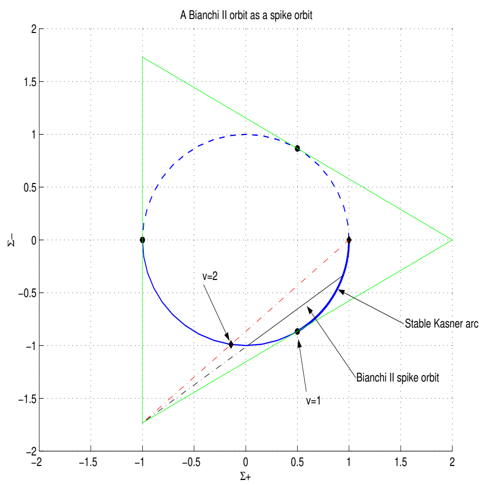
Taub orbit (subcase of Bianchi II)



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Kasner circle and stable arc

- The Kasner circle $\mathcal{K}$, is $\Sigma_+^2 + \Sigma_-^2 = 1$ is the fixed point set of the SB dynamics, and contains all limit states for the SB system.
- The rear $\Sigma_- < 0$ part of the Kasner circle is unstable under the Kasner dynamics. The stable arc $\mathrm{arc}(T_2, Q_1)$ is the set of stable fixed points for the SB system.
- The stable arc $\mathrm{arc}(T_2, Q_1)$ is a local attractor is the set of stable fix points. It connects the special point Q_1 with $v = 0$, to the point T_1 with $v = 1$. The Taub orbits with end points on $\mathrm{arc}(T_2, Q_1)$ originate on the Kasner circle with $v \in (1, 2)$.
- The "small data" result of Ringström deals with data close to the $\mathrm{arc}(T_2, Q_1)$.

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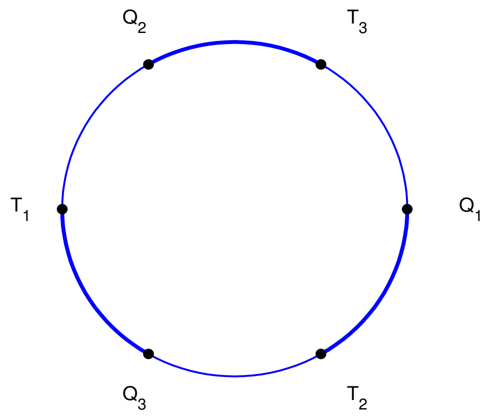
Kasner circle with special points



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Description in H-normalized variables

- Expect Gowdy to have non–oscillatory singularity with all limit states contained in $\mathcal{K}$.
- $\dot{U}, N_\times$ become uniformly small as t increases.
- Σ^2 close to 1 except along timelines close to spike points.
- $\Sigma_\times$ and N_- do not become uniformly small individually, but numerics indicates that their product $\Sigma_\times N_-$ does become uniformly small; hence expect that $\mathcal{U} = \{\Sigma_\times N_- = \Sigma_- N_\times = 0\}$ is the *uniform attractor* for Gowdy. $\mathcal{U}$ consists of the union of the Kasner and Taub spheres, which are joined at the Kasner circle.
- Combination of Kasner and Taub dynamics explains main features of Gowdy dynamics.
- timeline $t \mapsto \boldsymbol{X}(t, x)$ shadows orbit of SB system.

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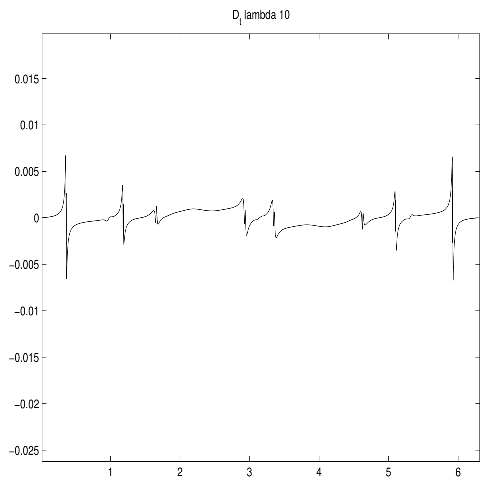
$N_-\Sigma_\times$ at $t = 10$



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Snapshots of Gowdy dynamics in H-normalized variables

- Next look at snapshots of Gowdy dynamics with Berger initial data.
- The solution approaches $\mathrm{arc}(T_2, Q_1)$, except for the spike timelines. We see both false (frame transition) and true (curvature transition) spikes.

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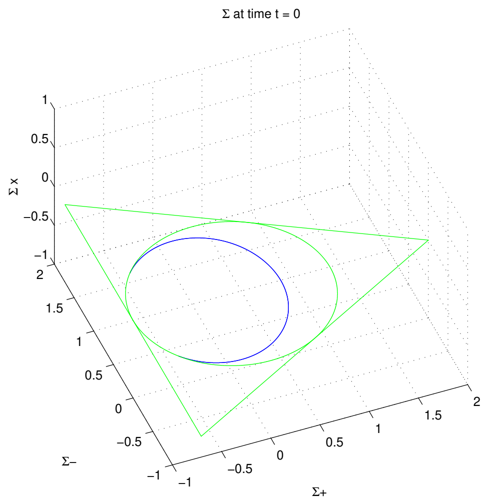
3d view of Σ at $t = 0$



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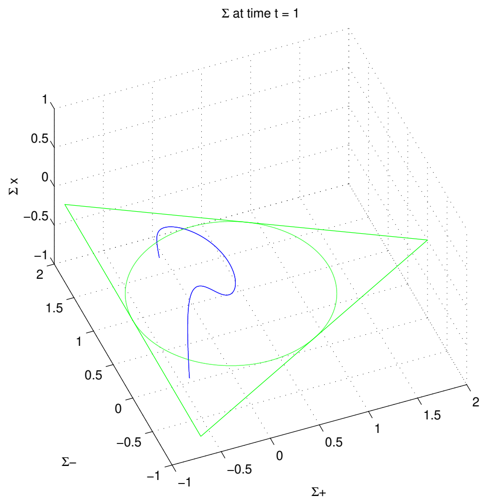
3d view of Σ at $t = 1$



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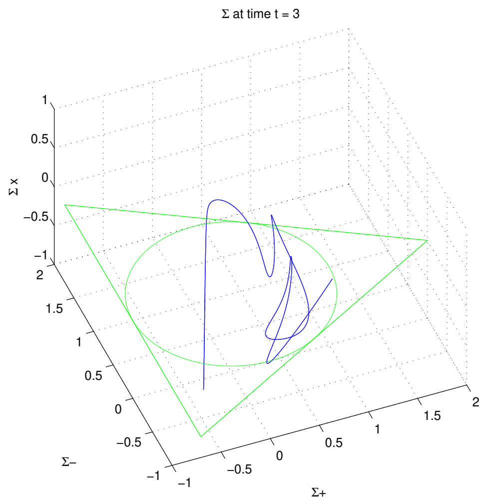
3d view of Σ at $t = 3$



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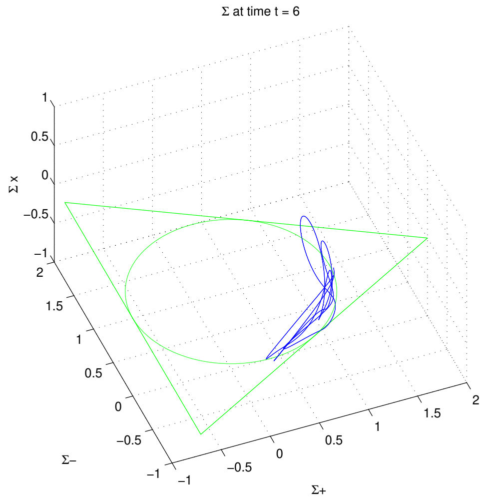
3d view of Σ at $t = 6$



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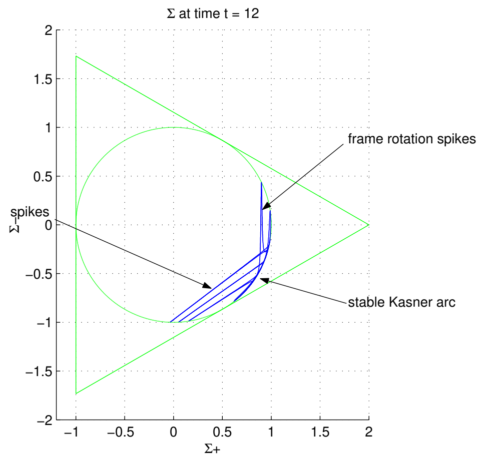
2d view of (Σ_+, Σ_-) at $t = 12$



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Gowdy dynamics shadows SB dynamics

Next look at example of orbit of the SB system and compare to timeline of full Gowdy system.

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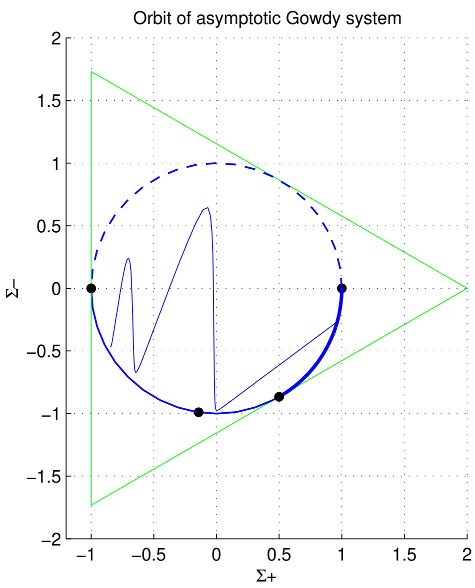

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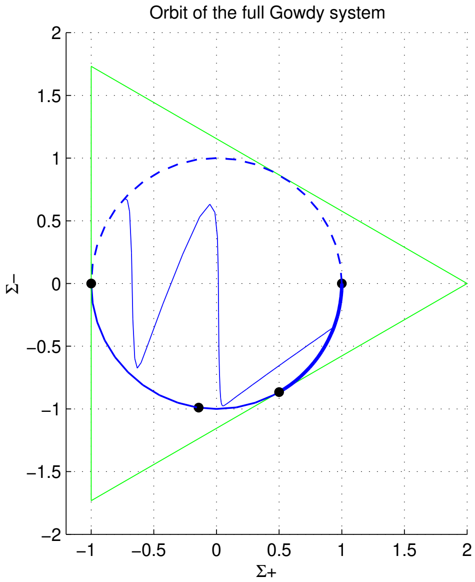

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Asymptotic velocity and Weyl curvature

Let k be the signed asymptotic velocity,

$$k(x) := -\frac{\sqrt{3}\hat{\Sigma}_-(x)}{1+\hat{\Sigma}_+(x)} ,$$

so that $\hat{v} = |k|$. Then the asymptotically nonvanishing Weyl curvature components are

$$\mathcal{E}_+ = \frac{6\,[\,1-k^2(x)\,]}{[\,3+k^2(x)\,]^2} + \mathcal{O}(e^{-t}) + \mathcal{O}(e^{-2[1-k(x)]t}) + \mathcal{O}(e^{-2k(x)t}) ,$$

$$\mathcal{E}_- = \frac{2\sqrt{3}k\,[\,1-k^2(x)\,]}{[\,3+k^2(x)\,]^2} + \mathcal{O}(e^{-t}) + \mathcal{O}(e^{-2[1-k(x)]t}) .$$

The asymptotically nonvanishing Weyl curvature scalars are

$$\mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta} = 6(\mathcal{E}_+^2 + \mathcal{E}_-^2 + \mathcal{E}_\times^2) ,$$

$$\mathcal{E}_\alpha{}^\beta\mathcal{E}_\beta{}^\gamma\mathcal{E}_\gamma{}^\alpha = -\,6\mathcal{E}_+\,(\mathcal{E}_+^2 - 3\mathcal{E}_-^2 - 3\mathcal{E}_\times^2) .$$

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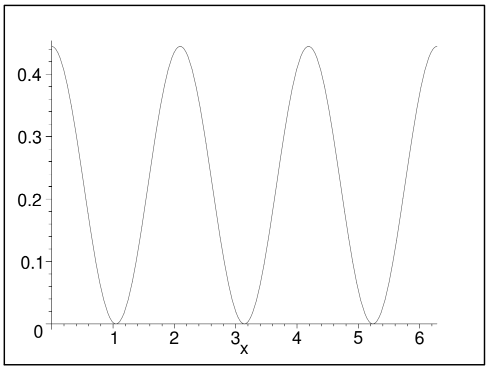
The curvature scalar $\mathcal{E}_{\alpha\beta}\mathcal{E}^{\alpha\beta}$



Note $2\pi/3$ periodicity.

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Concluding remarks

- In Bianchi IX, the Bianchi II attractor explains the Kasner billiard like behavior.
- The only Bianchi system with G_2 symmetry and nonzero twist is the exceptional Bianchi $\mathrm{VI}_{-1/9}$. Expected to have oscillatory singularity.
- Expect general G_2 to have oscillatory singularity (Berger et al., 2001).
- Expect general 3+1 spacetimes with nonstiff matter to have spacelike, local and oscillatory singularities (BKL proposal) [Berger et al., 2001].
- A candidate attractor variety has been identified for the general G_0 system [Uggla et al, gr-qc/0304002].
- A natural next step is to generalize the approach described here to vacuum G_2, and to investigate the G_2 attractor.

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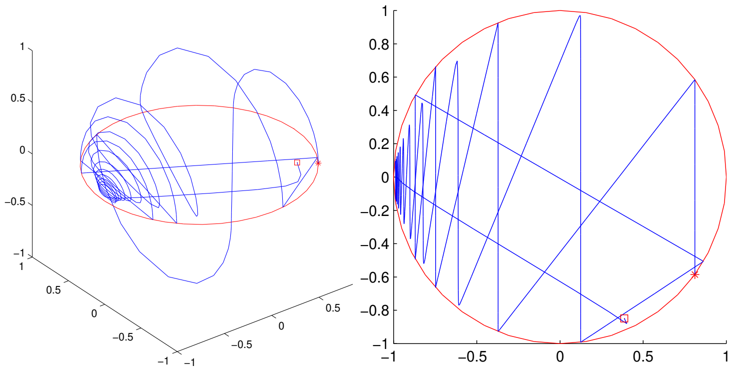
The exceptional Bianchi VI$_{-1/9}$ system



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