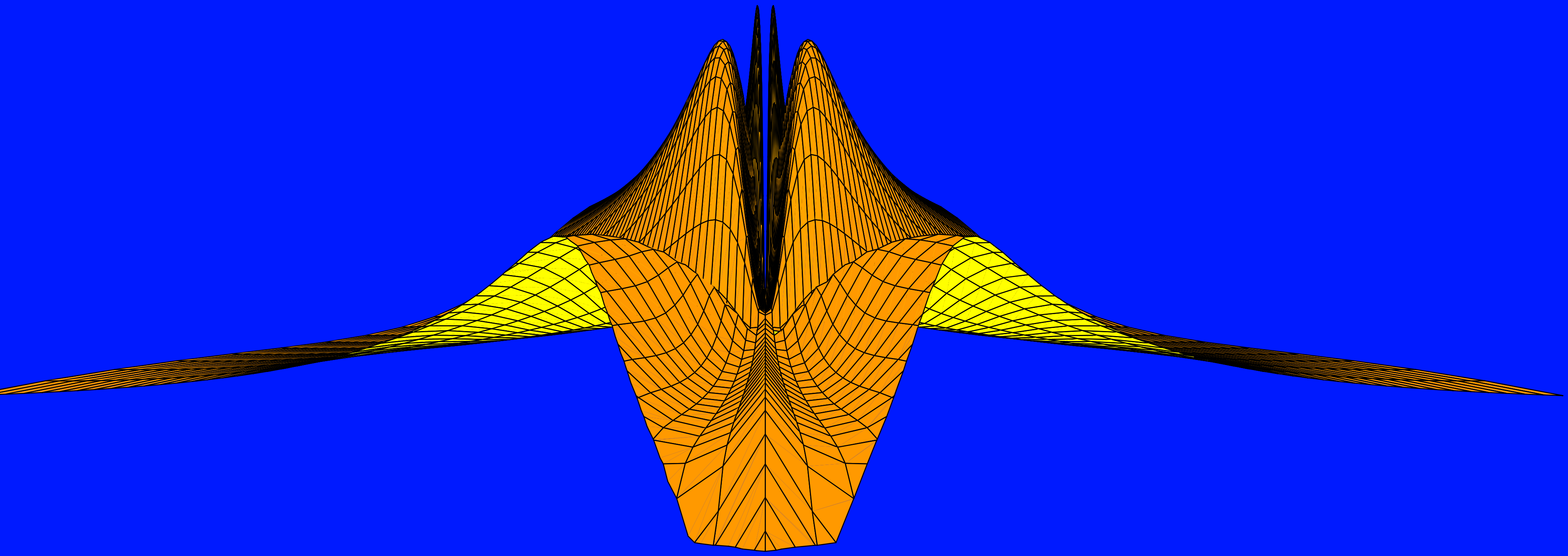

GLOBAL STRUCTURE OF THE CHOPTUIK SPACETIME

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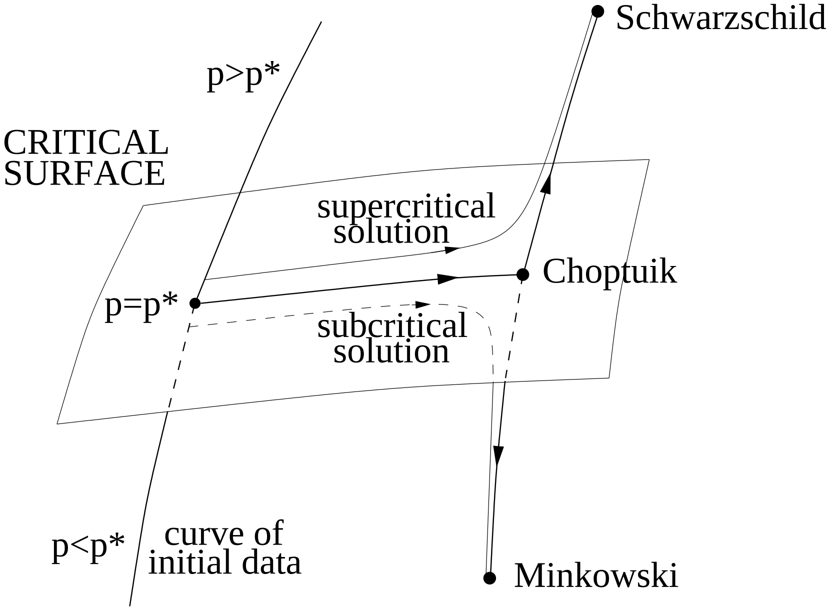
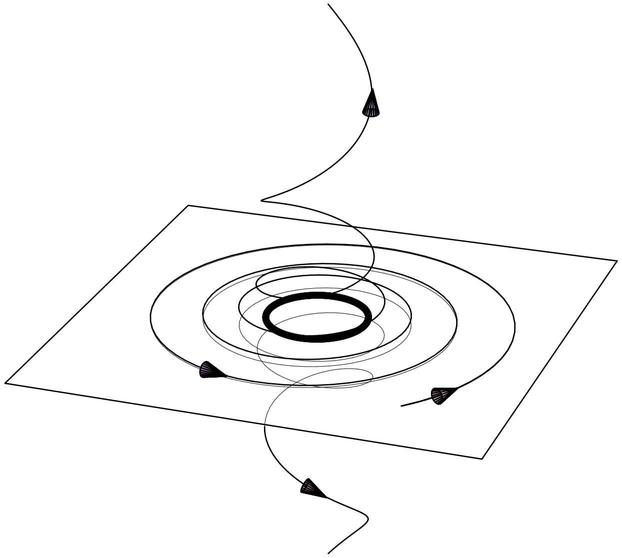
José M. Martín-García **IMAFF, CSIC, Madrid**

Introduction:

- Naked singularity in realistic gravitational collapse? Matter and initial conditions.
- Penrose 1969: cosmic censorship conjecture: no.
- Choptuik 1993: critical collapse: yes, with fine tuning. Mechanism:





Outline:

1. Possible structures for a self-similar spacetime.
2. Numerical construction of the Choptuik spacetime.
3. Interpretation of results.

1. Structure of spherical and self-similar spacetimes

1) **Self-similarity:** Scale invariance.

- Continous (CSS): Homothetic spacetime:

$$\mathcal{L}_\xi g_{ab} = -2 g_{ab}.$$

- Discrete (DSS):

$$\Phi_* g_{ab} = e^{-2\Delta} g_{ab}.$$

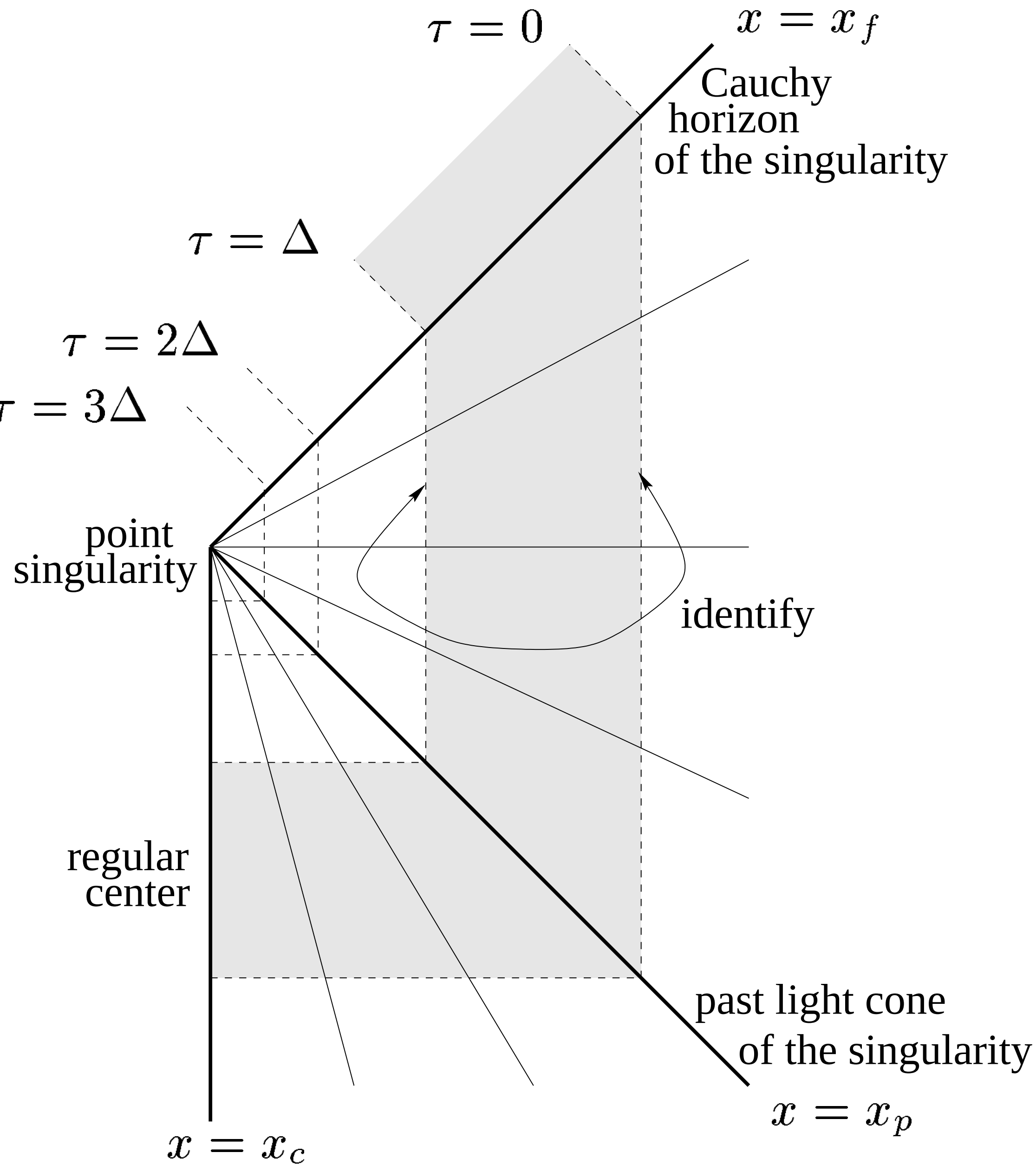
2) **Spherical symmetry:**

$$ds^2 = -\alpha^2(t,r)dt^2 + 2\beta(t,r)dtdr + a^2(t,r)dr^2 + R^2(t,r)d\Omega^2.$$

1)+2) **Adapted coordinates:**

$$ds^2 = e^{-2\tau}\left[A(\tau,x)d\tau^2 + 2B(\tau,x)d\tau dx + C(\tau,x)dx^2 + F^2(\tau,x)d\Omega^2\right].$$

CSS: Independence on τ,   DSS: Periodicity Δ on τ.



Central singularity.

Dimensional analysis:

$$R(\tau, x) \sim e^{2\tau}\bar{R}(\tau, x).$$

$$M(\tau, x) \sim e^{-\tau}\bar{M}(\tau, x).$$

Self-similarity horizons (SSH):

1. Null geodesics

2. Invariance under self-similarity

Sketch of argument (C. Gundlach and JMM, Phys. Rev. D 2003)

1) Radial null geodesics:

$$ds^2 = 0 = A d\tau^2 + 2B d\tau dx + C dx^2.$$

Two decoupled ODEs:

$$\frac{dx(\tau)}{d\tau} = f_\pm[x(\tau), \tau].$$

2) SSH: $dx = 0$**, equivalent to** $A = 0$**.**

3) SSHs are attractors or repellers of null geodesics.

4) Coordinate change to $x = 0$ **for the SSH and** $C = 0$**. Expand metric around** $x = 0$**:**

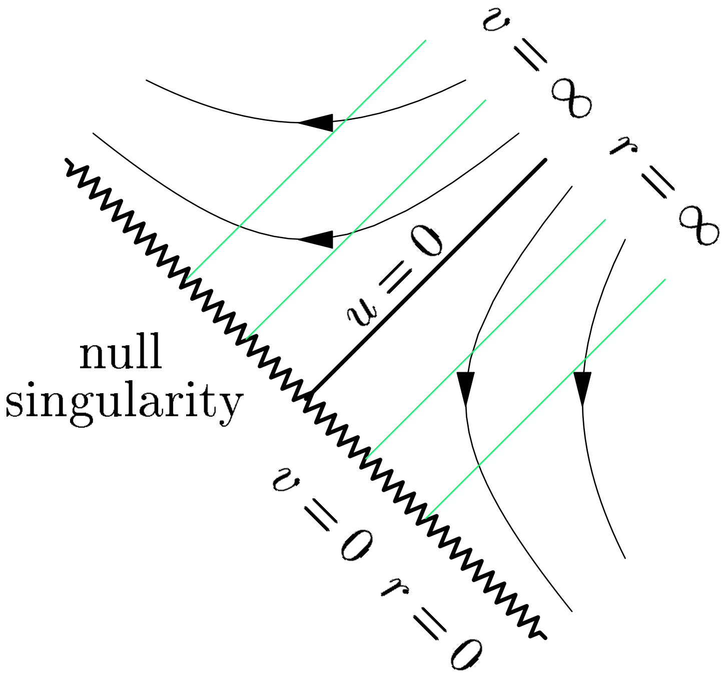
$$\begin{aligned} ds^2 &= e^{-2\tau}\left[4qx d\tau^2 + 2d\tau dx + f^2(\tau) d\Omega^2\right] + \text{higher orders} \\ &= -du\, dv + f^2(\tau) v^{1/(1+q)} d\Omega^2, \end{aligned}$$

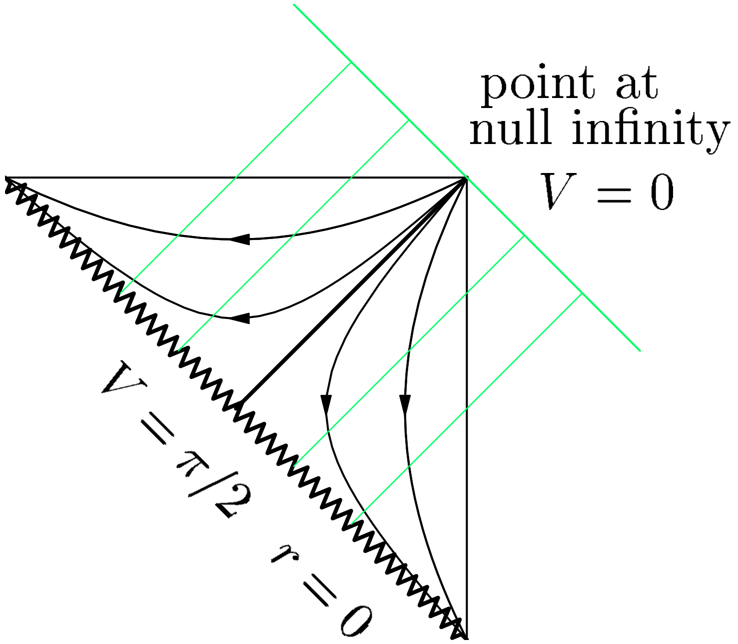
with $u = x\, e^{2q\tau}/(1+q)$**,** $v = e^{-2(1+q)\tau}$**.**

5) Curvature:

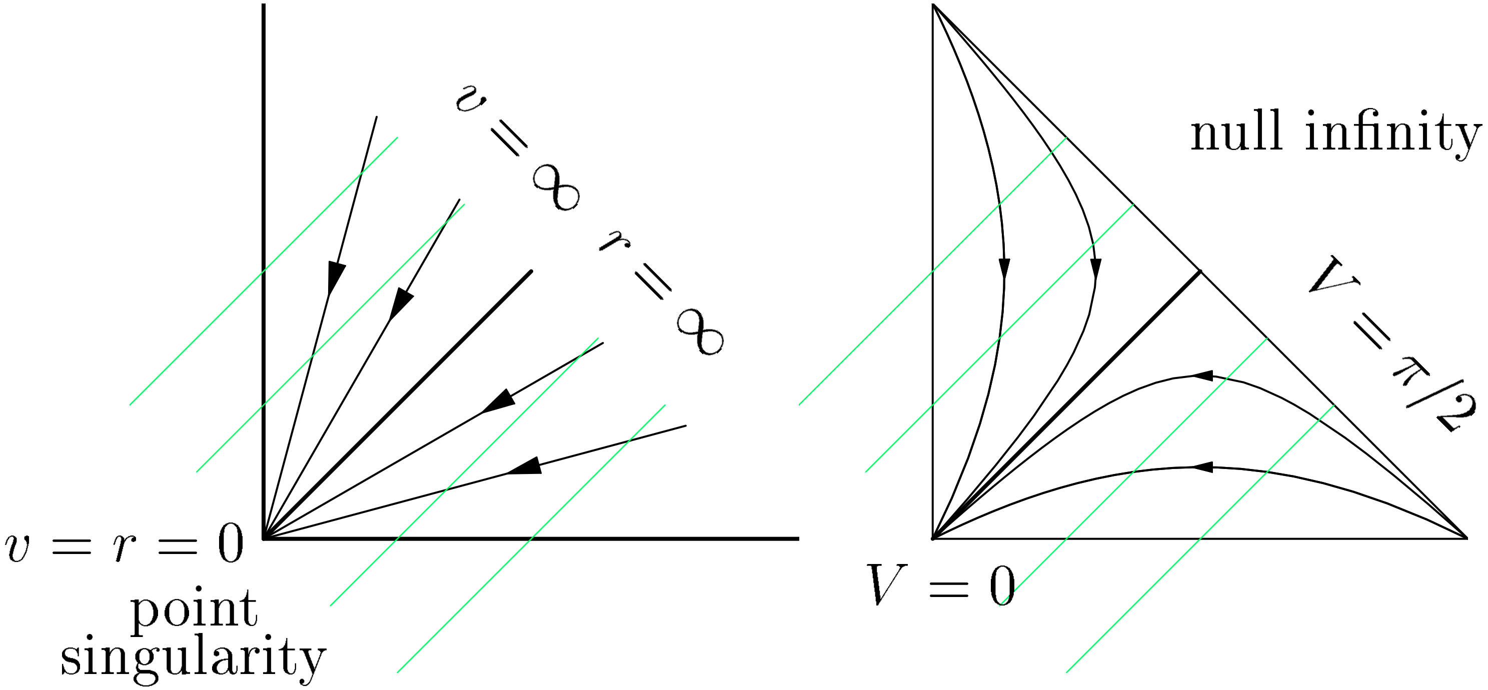
$$R_{22} = \frac{1+2q}{2(1+q)^2}\frac{1}{v^2}, \qquad R_{33} = R_{44} = \frac{1}{r^2}, \qquad R = \frac{2}{r^2}.$$

$q > 0$, Splash

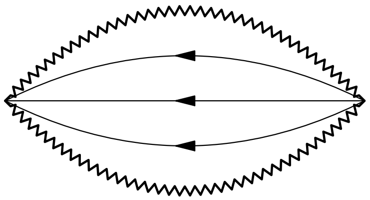

$v = \infty$ $r = \infty$
$u = 0$
null singularity
$v = 0$ $r = 0$

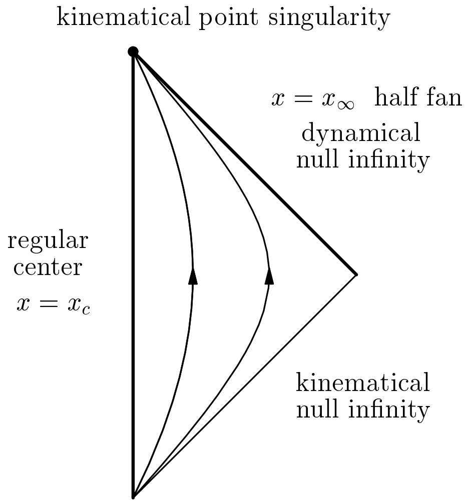

point at null infinity
$V = 0$
$V = \pi/2$
$r = 0$

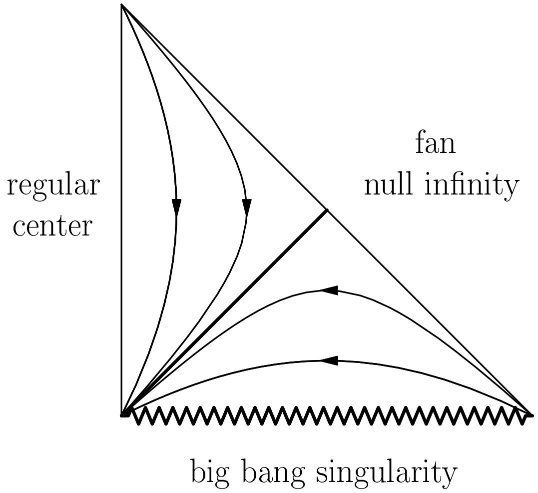
$-1 < q < 0$, Regular Fan

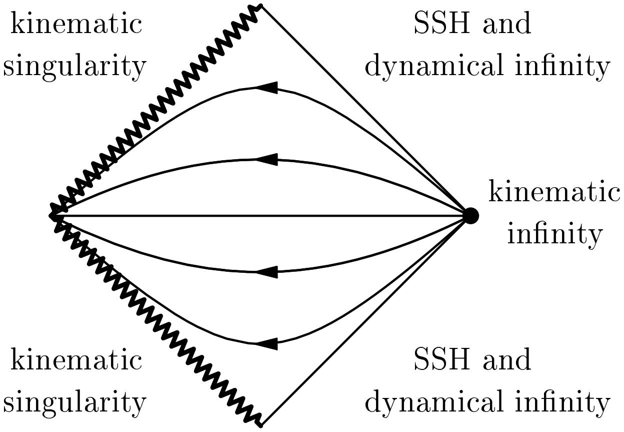


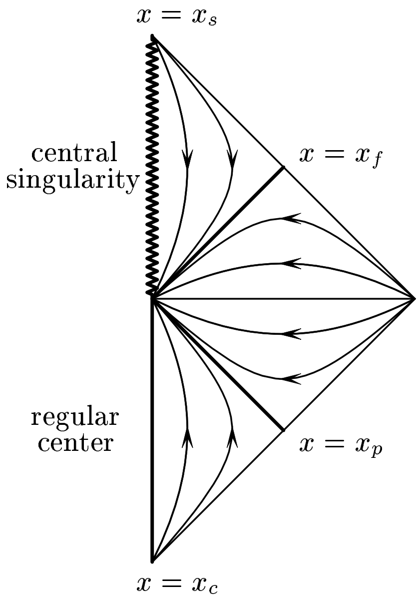
Examples

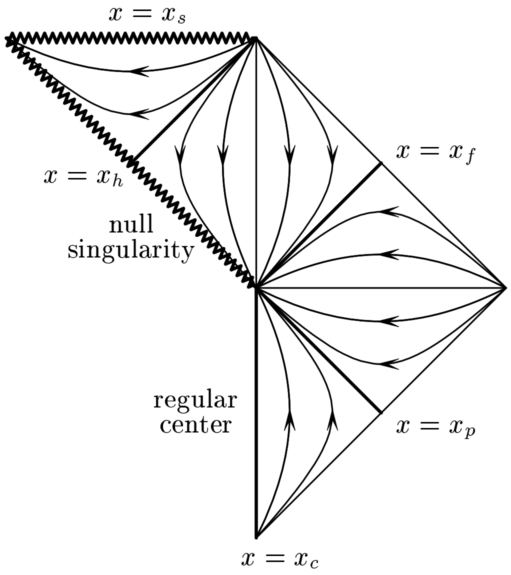



kinematical point singularity
$x = x_\infty$ half fan
dynamical null infinity
regular center
$x = x_c$
kinematical null infinity


regular center
fan null infinity
big bang singularity


kinematic singularity
SSH and dynamical infinity
kinematic infinity
kinematic singularity
SSH and dynamical infinity


$x = x_s$
central singularity
$x = x_f$
regular center
$x = x_p$
$x = x_c$


$x = x_s$
$x = x_f$
$x = x_h$
null singularity
regular center
$x = x_p$
$x = x_c$

2. Construction of the Choptuik spacetime

(JMM and C. Gundlach, Phys. Rev. D 2003)

2.1. Definition of the solution

- **System**: Einstein + Klein-Gordon + spherical.

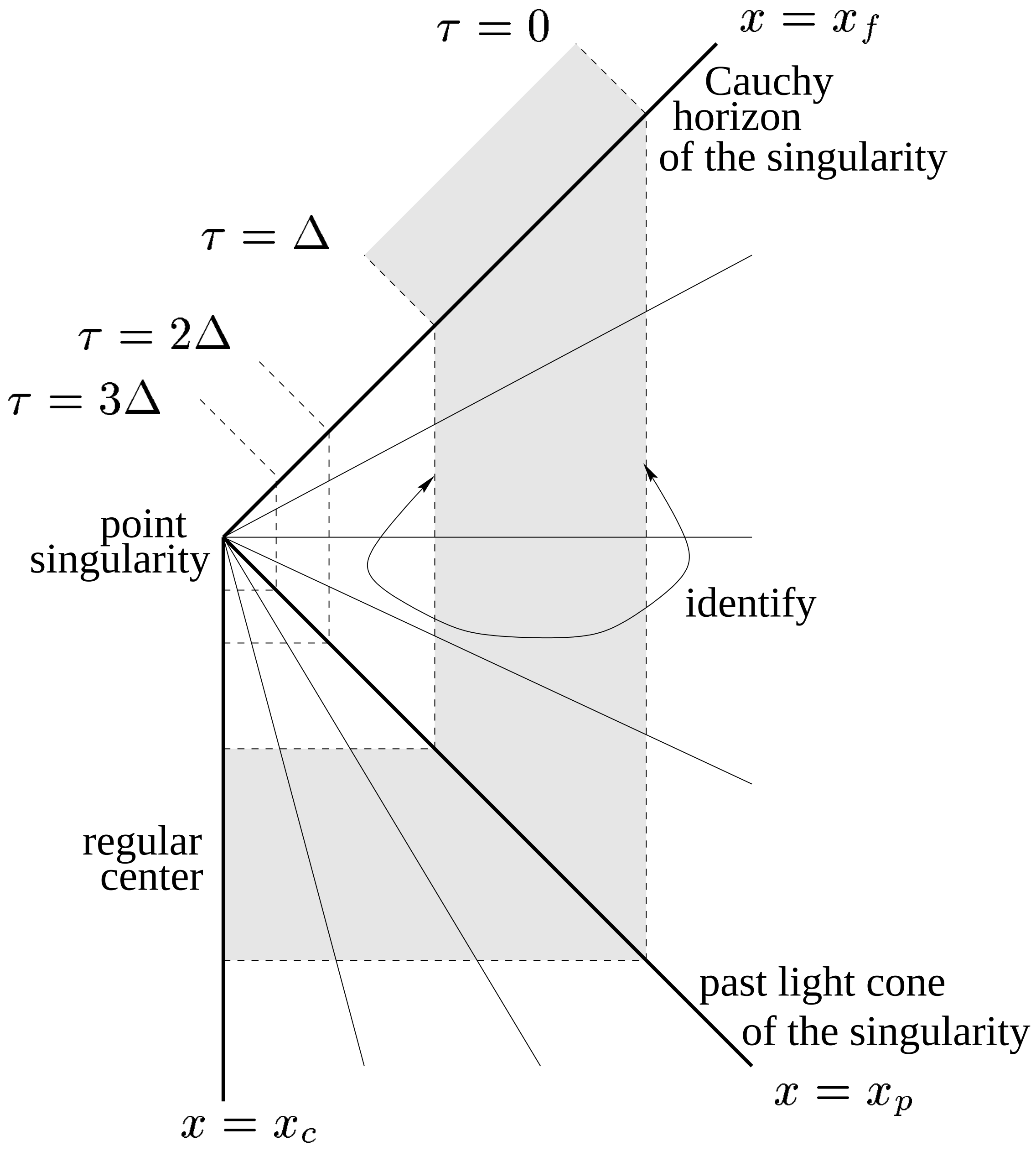
$$g^{ab}\,\nabla_a\,\nabla_b\,\phi = 0, \qquad R_{ab} = 8\pi G\,\nabla_a\,\phi\,\nabla_b\,\phi.$$

- **Scalar field**:

$$U = \sqrt{2\pi G}\,\nabla_u\phi, \qquad V = \sqrt{2\pi G}\,\nabla_v\phi, \qquad \textbf{with} \qquad \nabla_u r = -r, \quad \nabla_v r = r.$$

- **Spherical metric**: Functions A, B, C, F.
- **DSS**: A, B, C, F, U, V periodic in τ with period Δ.
- Generic solutions are singular at the center and at the light cones.
- **Choptuik spacetime**:
  - Additional symmetry: $A(\tau + \Delta/2, x) = A(\tau, x)$, $U(\tau + \Delta/2, x) = -U(\tau, x)$.
  - Regularity at the center and the past light cone.

$\Longrightarrow$ Nonlinear boundary value problem on the past patch (eigenvalue $\triangle$).


$\tau = 0$
$x = x_f$
Cauchy horizon of the singularity
$\tau = \Delta$
$\tau = 2\Delta$
$\tau = 3\Delta$
point singularity
identify
regular center
past light cone of the singularity
$x = x_p$
$x = x_c$

2.2. Solution on the past patch

- **Numerical methods:**
  - **pseudospectral method (128 Fourier modes per function)**
  - **double shooting to a fitting point. Newton's method.**
- **Gauge choice:**

$$\tau = -\log(-t), \quad x = r/(-t), \qquad \overline{ds}^2 = -a^2 f^2 d\tau^2 + a^2(dx - x\,d\tau)^2 + x^2 d\Omega^2.$$

- **Results:** $\Delta = 3.445\,452\,402(3).$